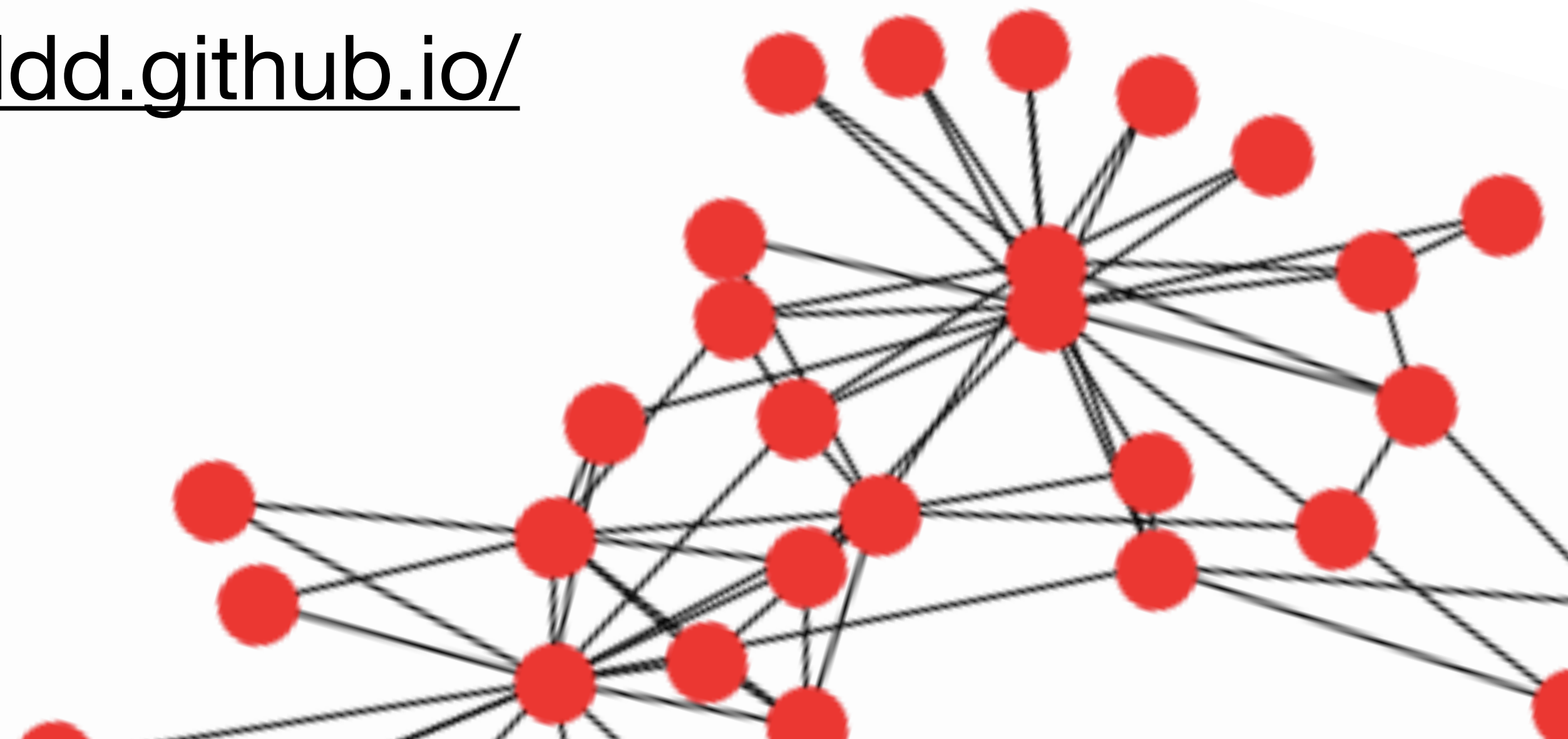


Week 9: Epidemics and mobility

Naomi Arnold

<https://narnolddd.github.io/>



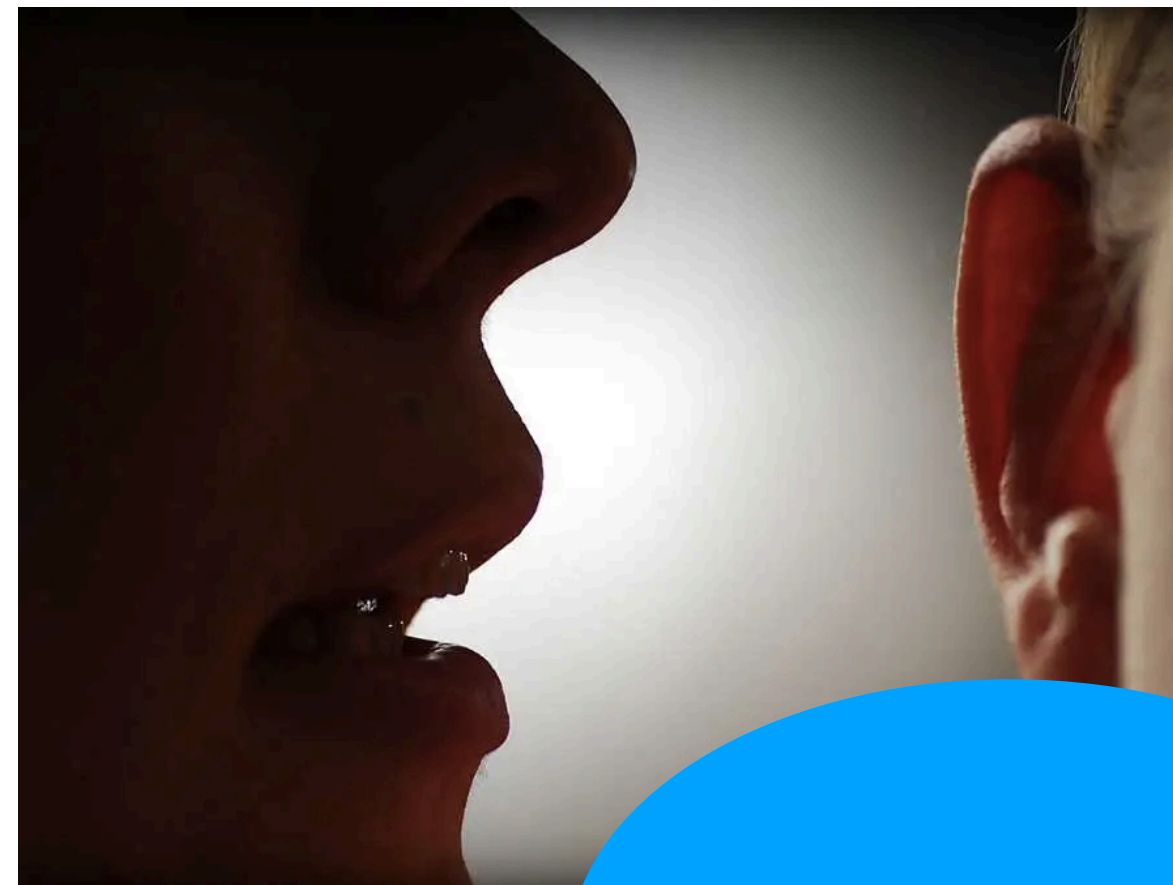
Tutorial aims

- Recap different **epidemic models** for networks
- Discuss different epidemic **intervention measures** in the context of networks and epidemic models
- Look at some **numerical simulations** implemented in Python

What might be modelled as an epidemic?



Infectious diseases



Information spread



Spread of memes

<https://covid19obs.fbk.eu/>

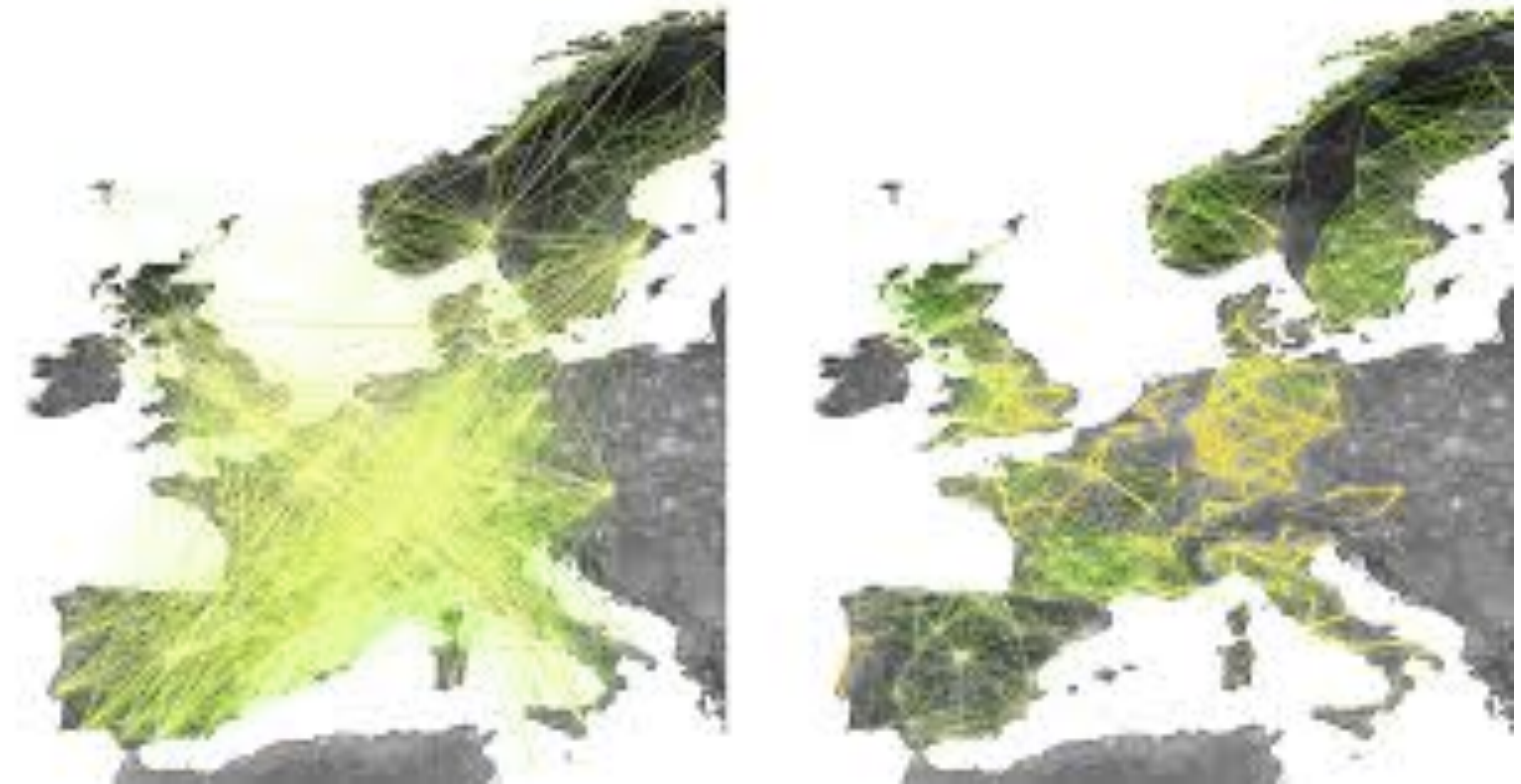
[Wang, L., Wood, B. C., An epidemiological approach to model the viral propagation of memes, Applied Mathematical Modelling, 2011]

[Weng, L., Flammini, A., Vespignani, A. & Menczer, F. Competition among memes in a world with limited attention, Scientific Reports, 2012]

Underlying network



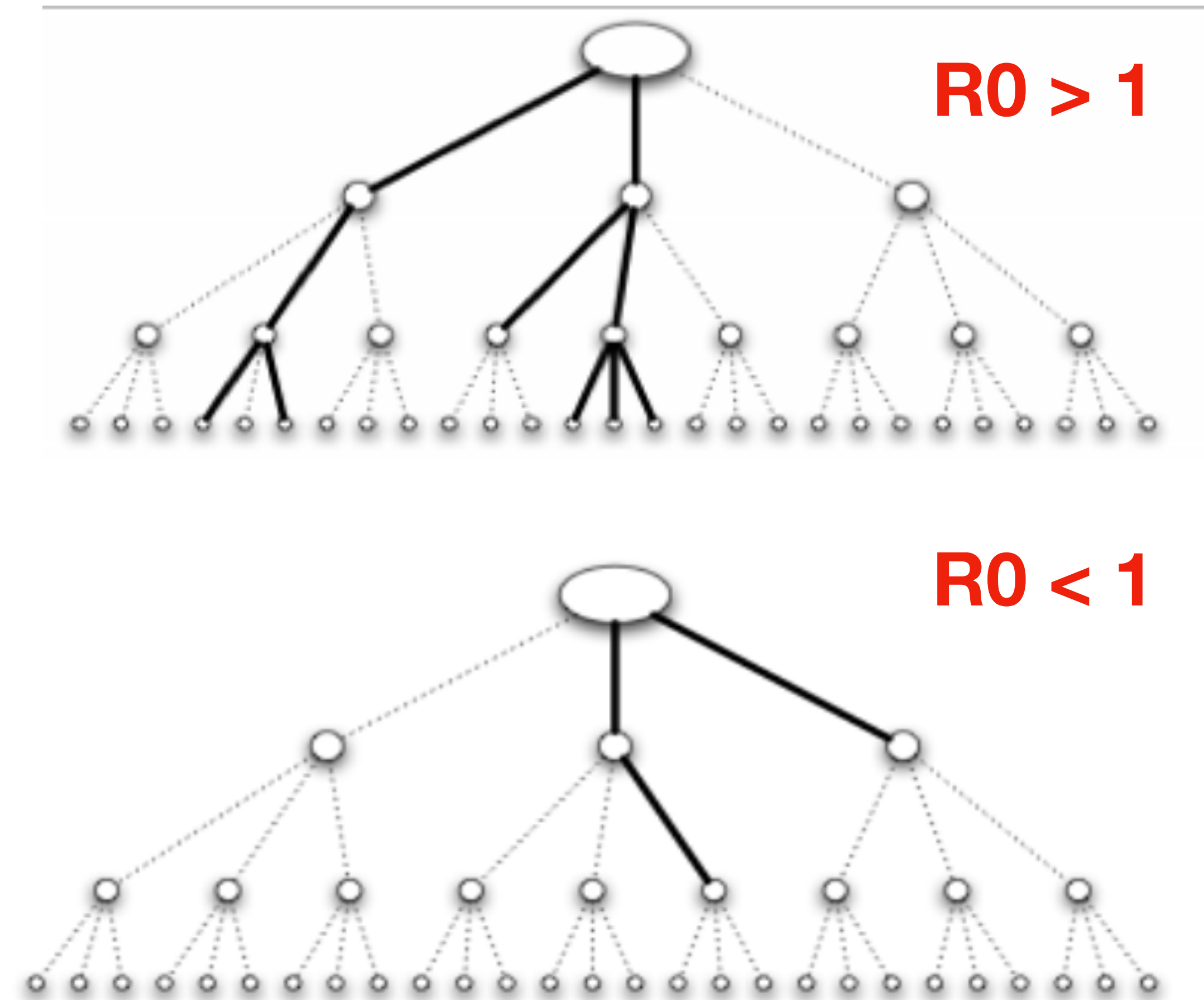
Face-to-face contact network



Airport network (providing more global picture picture)

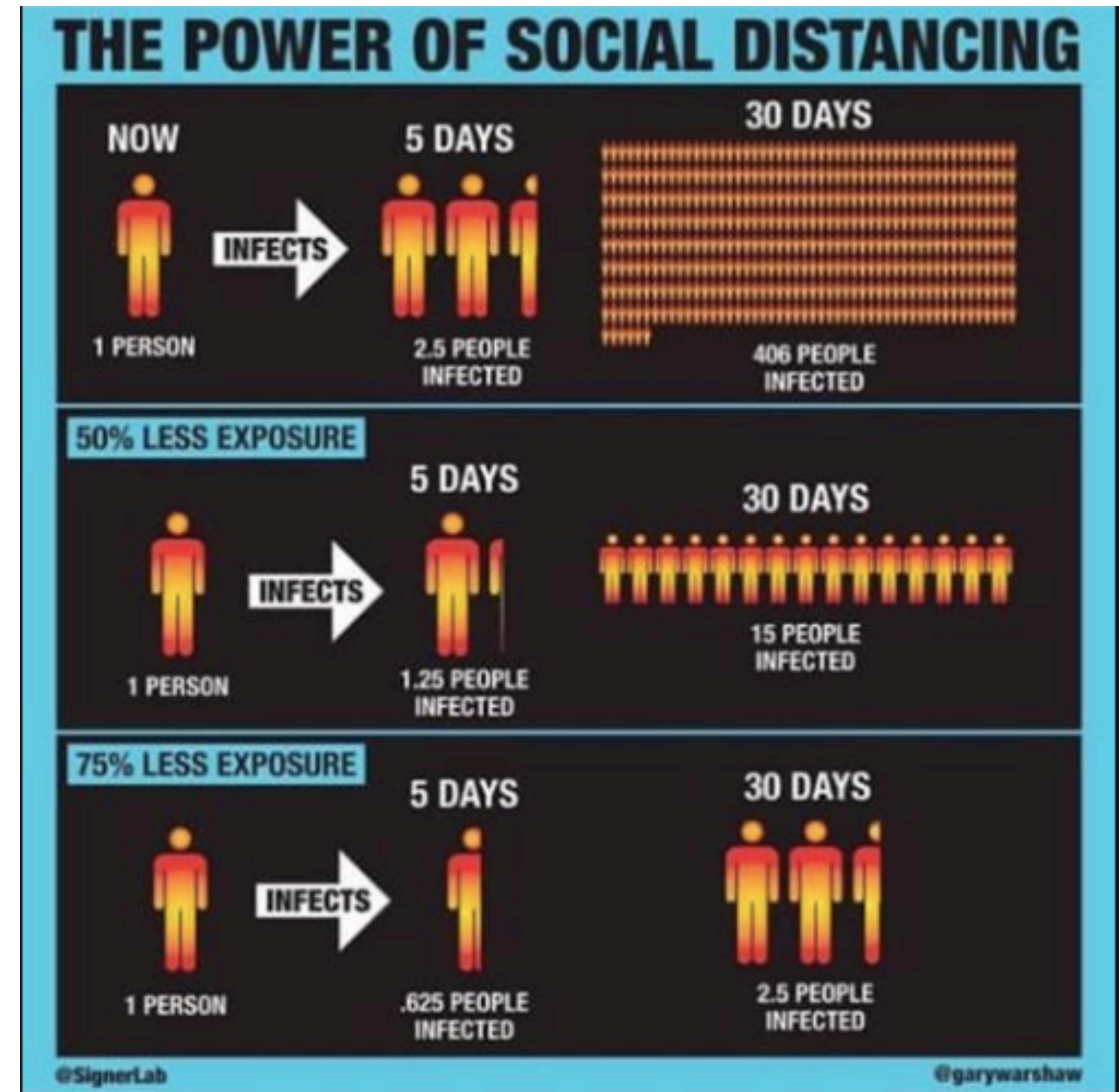
Simple Epidemic Model

- Start with single infected person in the population.
- **First wave:** Infected person meets k people, infecting each with probability p , so kp new infected individuals after this wave.
- $kp = R_0$ (basic reproductive number) — **number of cases** infected by one person



Simple Epidemic Model

- **Second wave:** each of these kp infected individuals goes on to meet k people, again infecting each with probability p .
- $kp \times kp = (kp)^2$ new infected individuals.
- Or in terms of R_0 , $(R_0)^2$ in second wave.



Example: Total # infected

Disease with $R_0 = 2$ using this model. Starting with one person infected, how many will be infected after 5 waves? (assuming individual **stays** infected)

**Initial
infected
person**

Example: Total # infected

Disease with $R_0 = 2$ using this model. Starting with one person infected, how many will be infected after 5 waves? (assuming individual **stays** infected)

1

Initial
infected
person

Example: Total # infected

Disease with $R_0 = 2$ using this model. Starting with one person infected, how many will be infected after 5 waves? (assuming individual **stays** infected)

$$1 + 2$$

Initial
infected
person

Two people
infected in
first wave



Example: Total # infected

Disease with $R_0 = 2$ using this model. Starting with one person infected, how many will be infected after 5 waves? (assuming individual **stays** infected)

$$1 + 2 + 2^2$$

Initial
infected
person

Two people
infected in
first wave

Each of those
infects **two**
more

Example: Total # infected

Disease with $R_0 = 2$ using this model. Starting with one person infected, how many will be infected after 5 waves? (assuming individual **stays** infected)

$$1 + 2 + 2^2 + 2^3 + 2^4 + 2^5$$

Initial
infected
person

Two people
infected in
first wave

Each of those
infects **two**
more

Subsequent waves

Example: Total # infected

Disease with $R_0 = 2$ using this model. Starting with one person infected, how many will be infected after 5 waves? (assuming individual **stays** infected)

$$1 + 2 + 2^2 + 2^3 + 2^4 + 2^5$$

Initial
infected
person

Two people
infected in
first wave

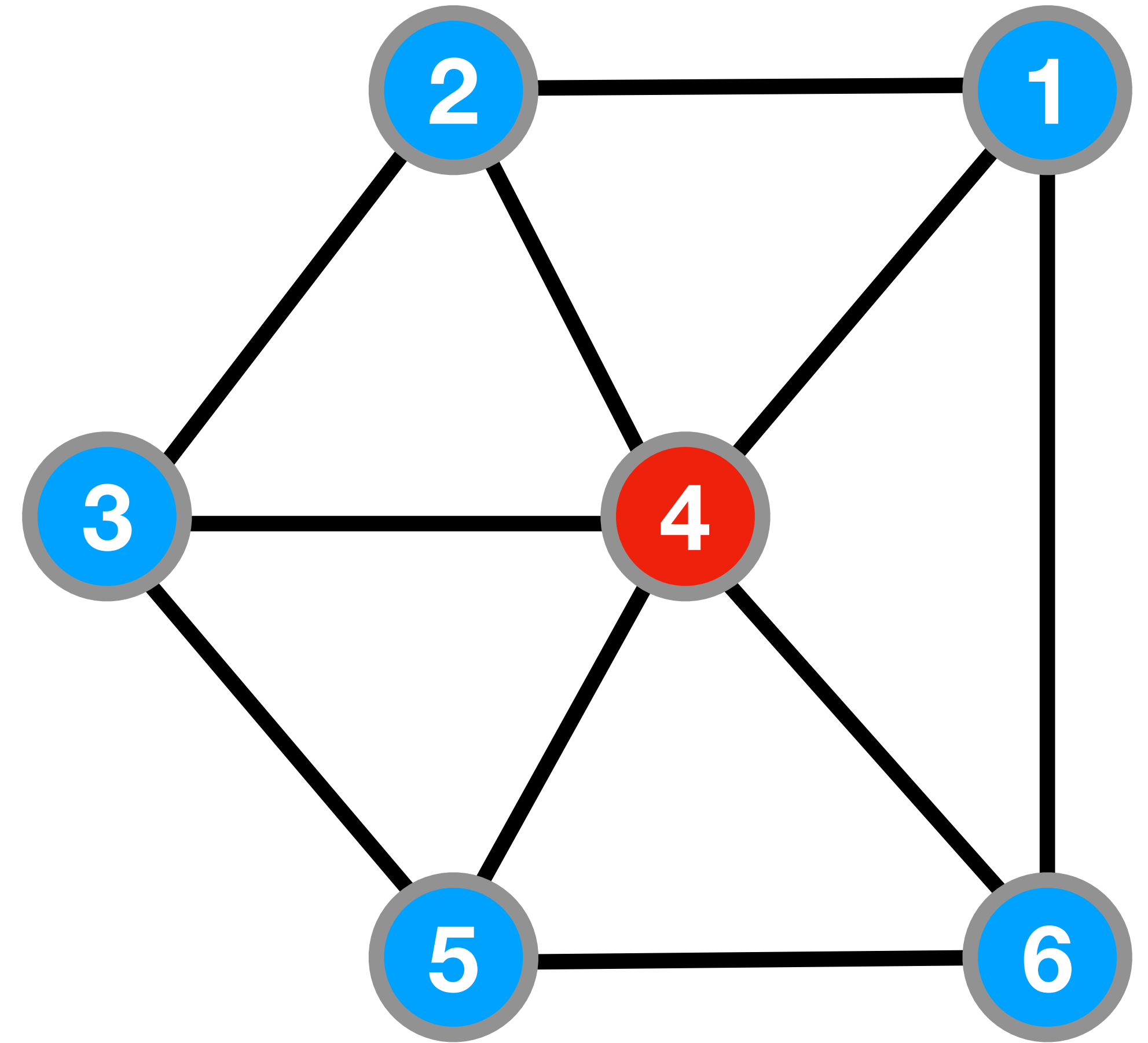
Each of those
infects **two**
more

Subsequent waves

Total infected: **63**

Epidemics on networks (SI)

- Nodes are either **susceptible (S)** or **infected (I)**. Once infected will never recover.
- An infected node infects its neighbours with a **rate β** .
- Ultimately the **whole network** will become infected (provided it's connected).



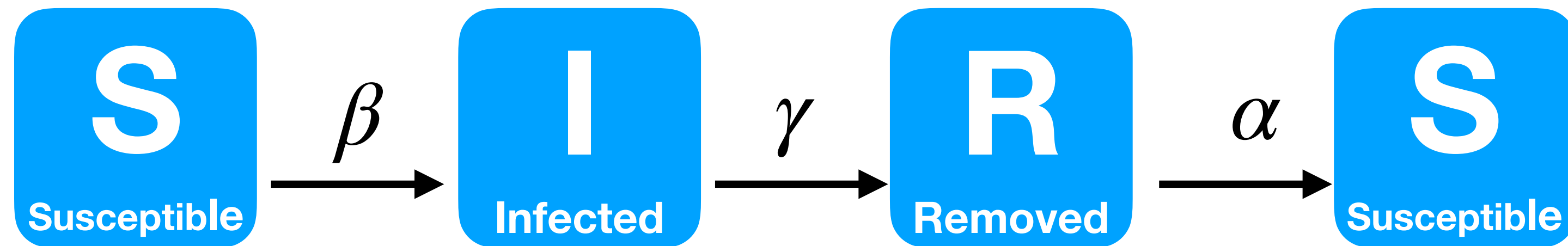
More realistic models



Node recovers and becomes **susceptible** again after being infected, so can be infected **multiple times**.



Nodes are **recovered/removed** after being infected. This means they are **immune** to the disease.

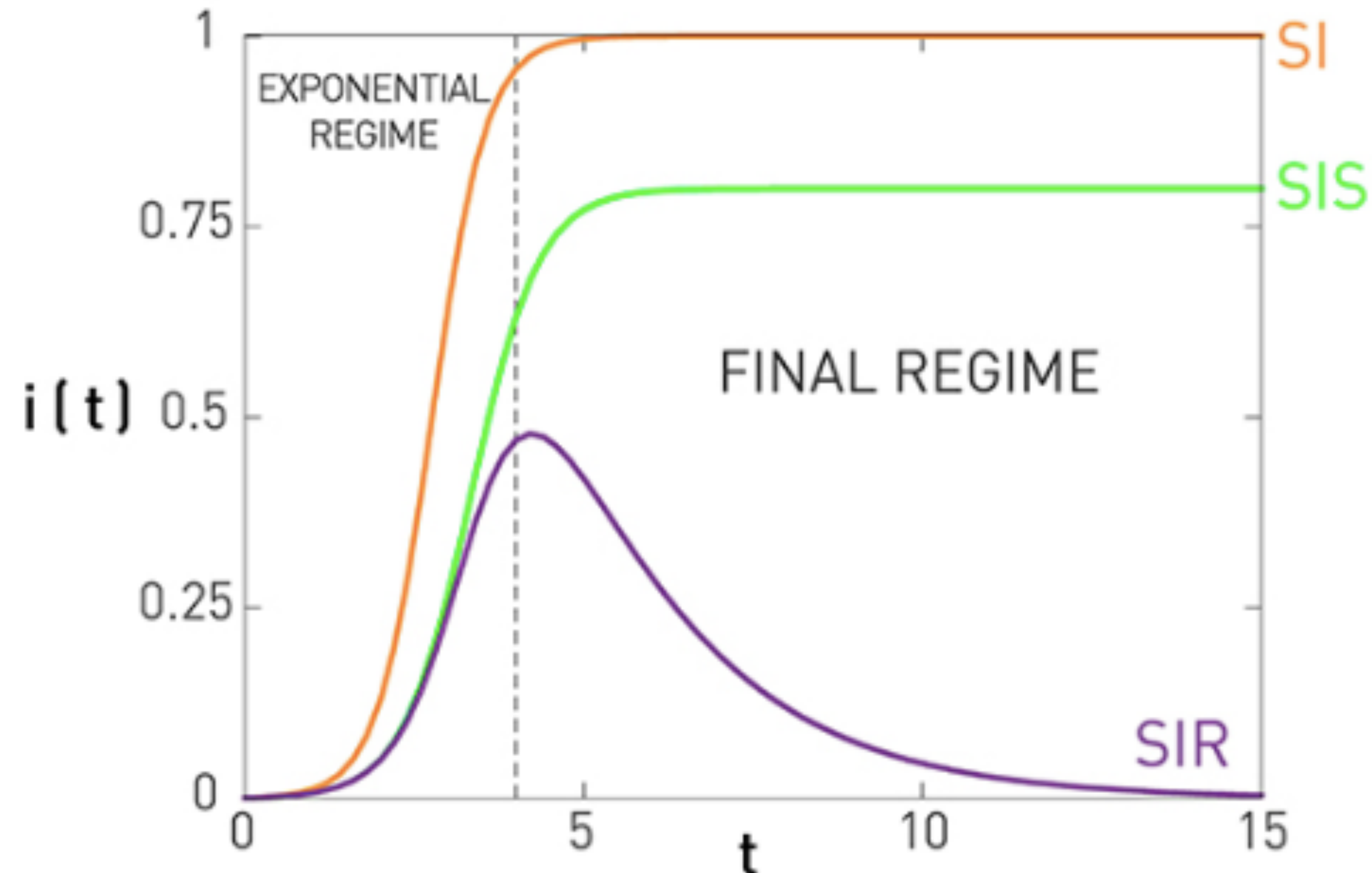


Nodes have **temporary immunity** after being infected

Which to use? Depends on the **disease** and the **application**...

Infection curves

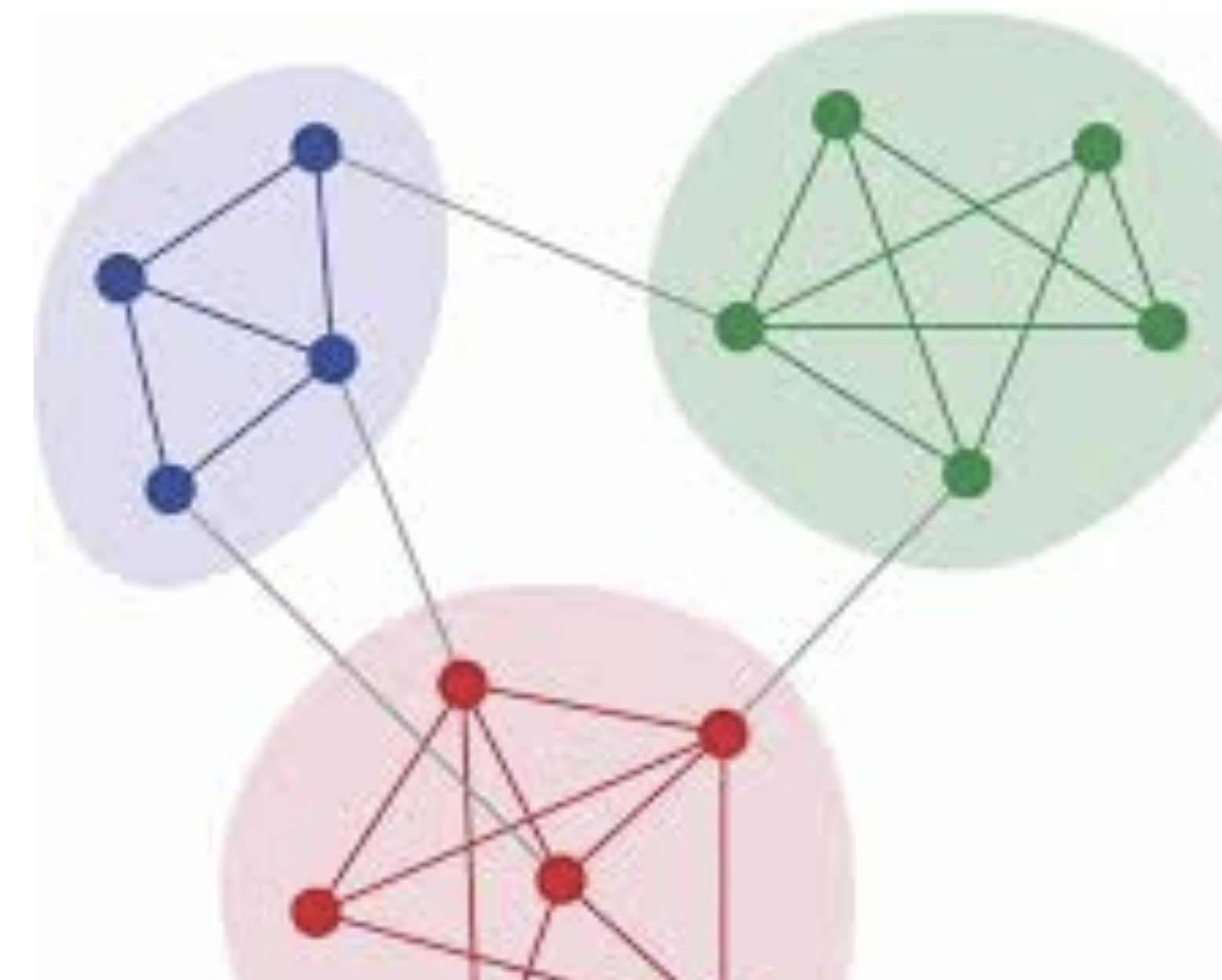
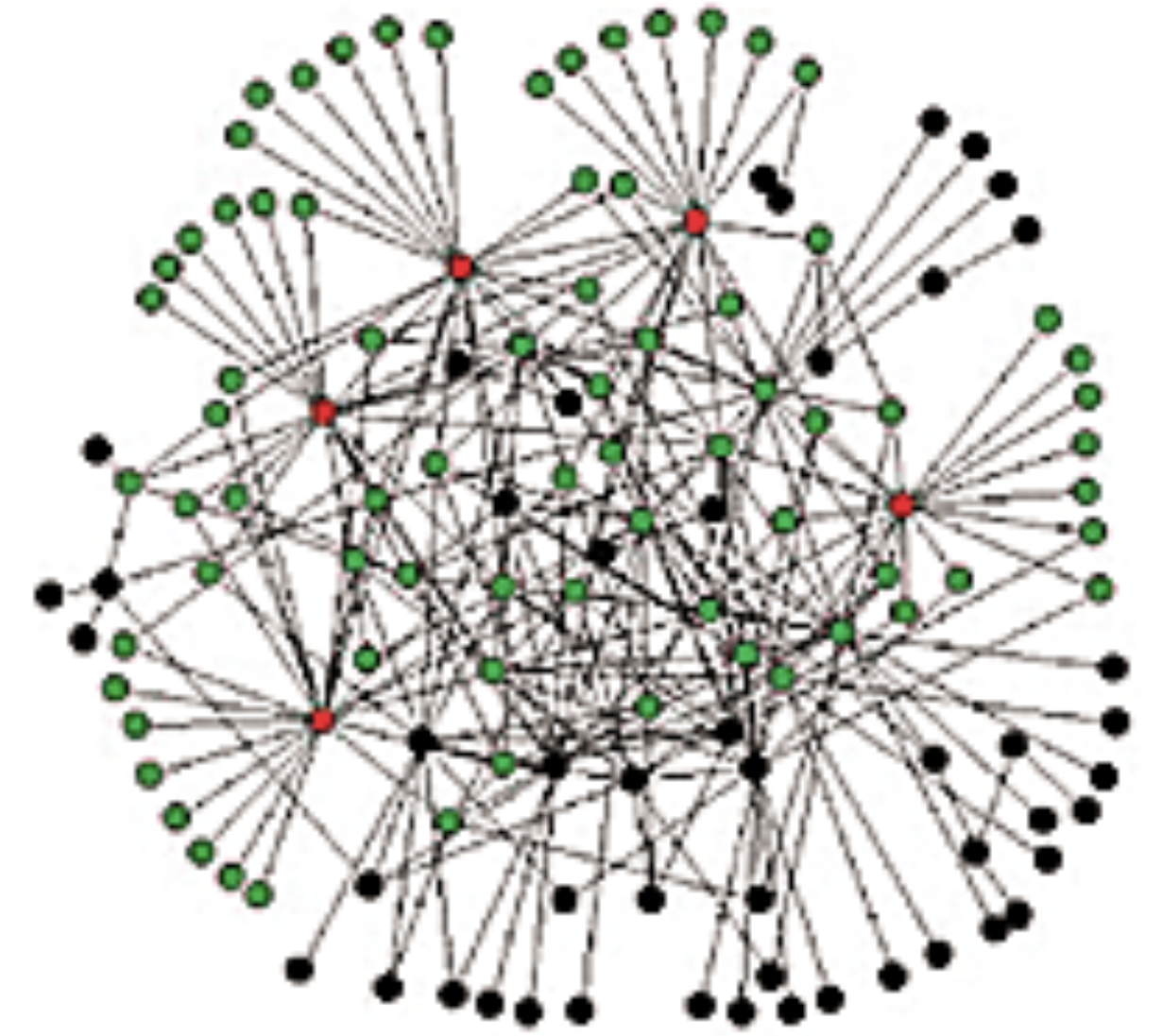
Plot of number of infected individuals over time for different models
(taken from Barasi's Network Science Book)



- **SI:** whole population becomes infected
- **SIS:** disease reaches endemic state, where a constant proportion of people infected
- **SIR:** disease hits a peak, after which enough people are immune that the disease dies out

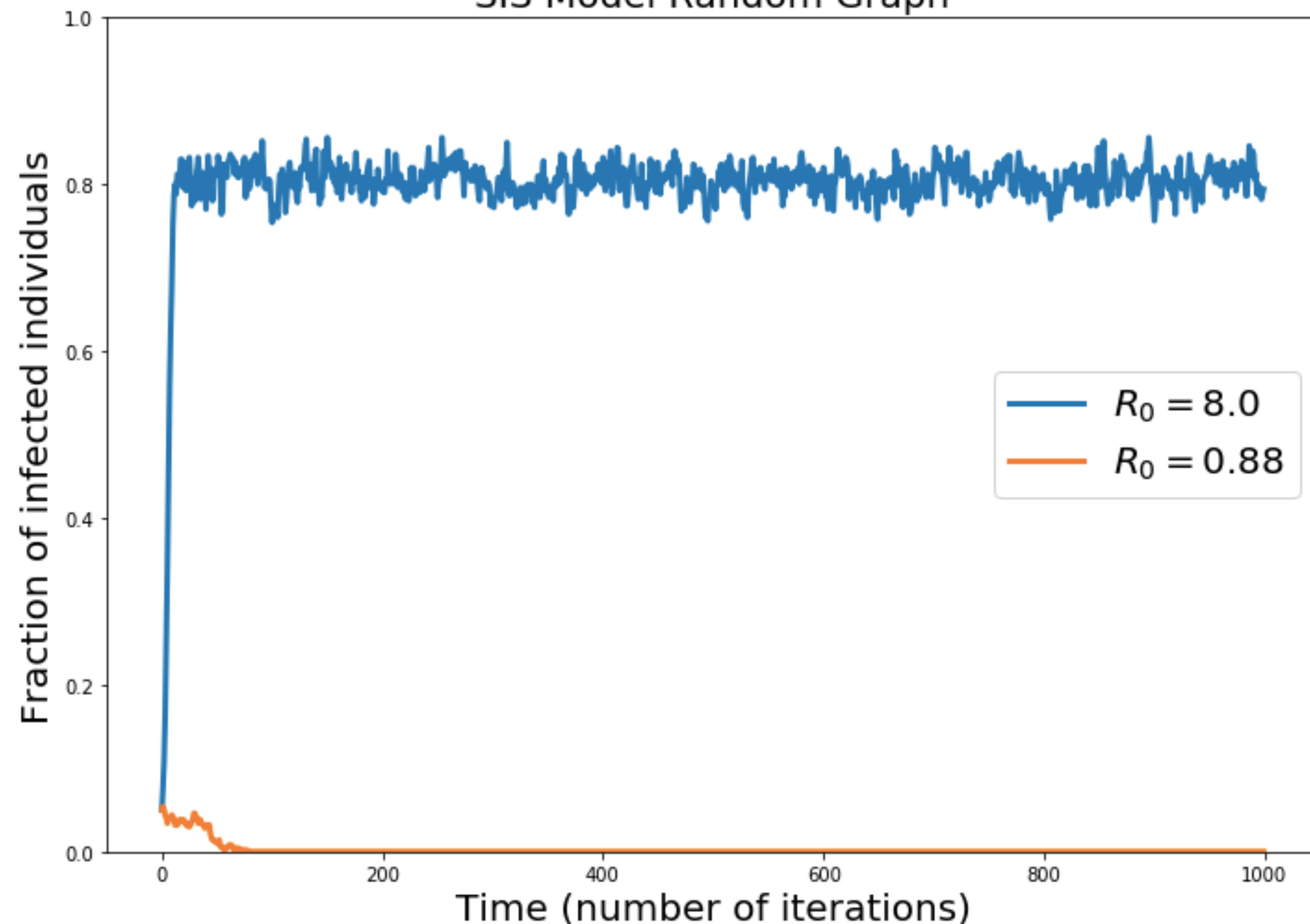
Role of network structure

- **Heterogeneous degree distribution** (e.g. scale-free networks, and often real social networks) can **speed up** the spread of diseases, and make them **persist** even if they have **low infection rate**
- Largely due to presence of **highly connected hubs** — perhaps (??) why we seem to have seen so many celebrities who have tested positive for COVID-19
- **Modular structure** (tightly knit communities with few links between) can help slow down spread



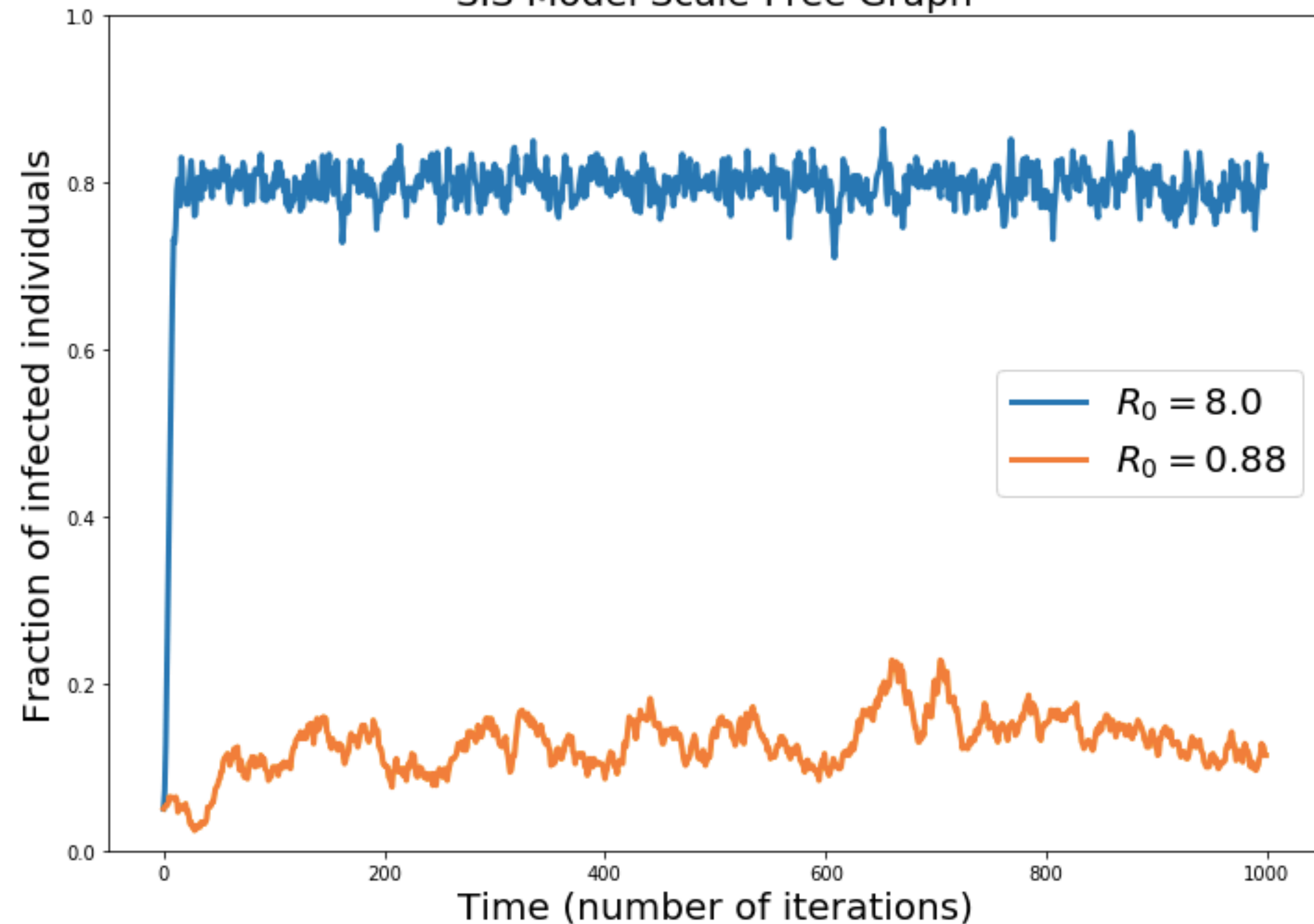
SIS: Random vs Scale-free

SIS Model Random Graph



Erdos-Renyi graph, disease with $R_0 < 1$ dies out quickly

SIS Model Scale-Free Graph



Scale free graph, same disease persists

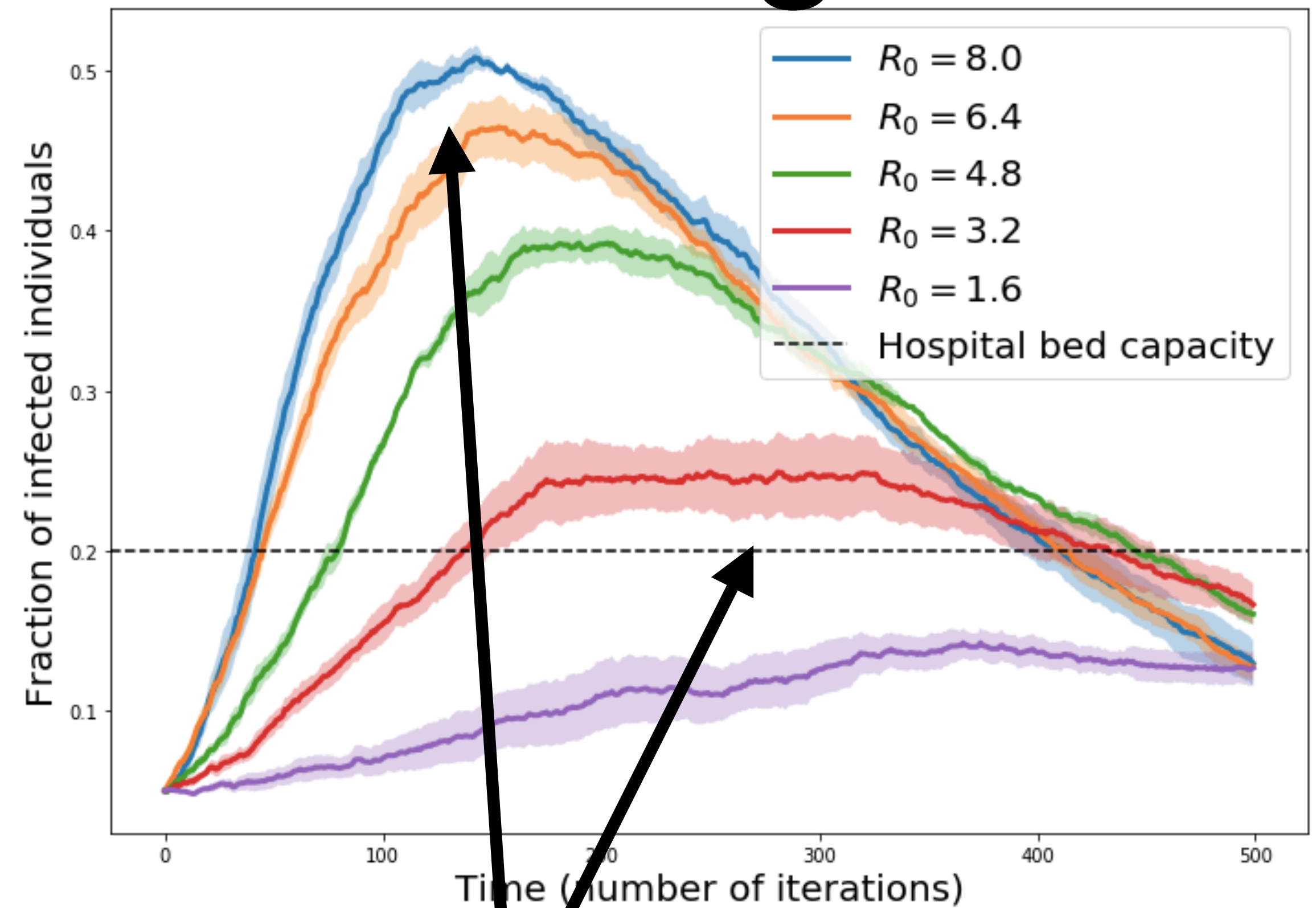
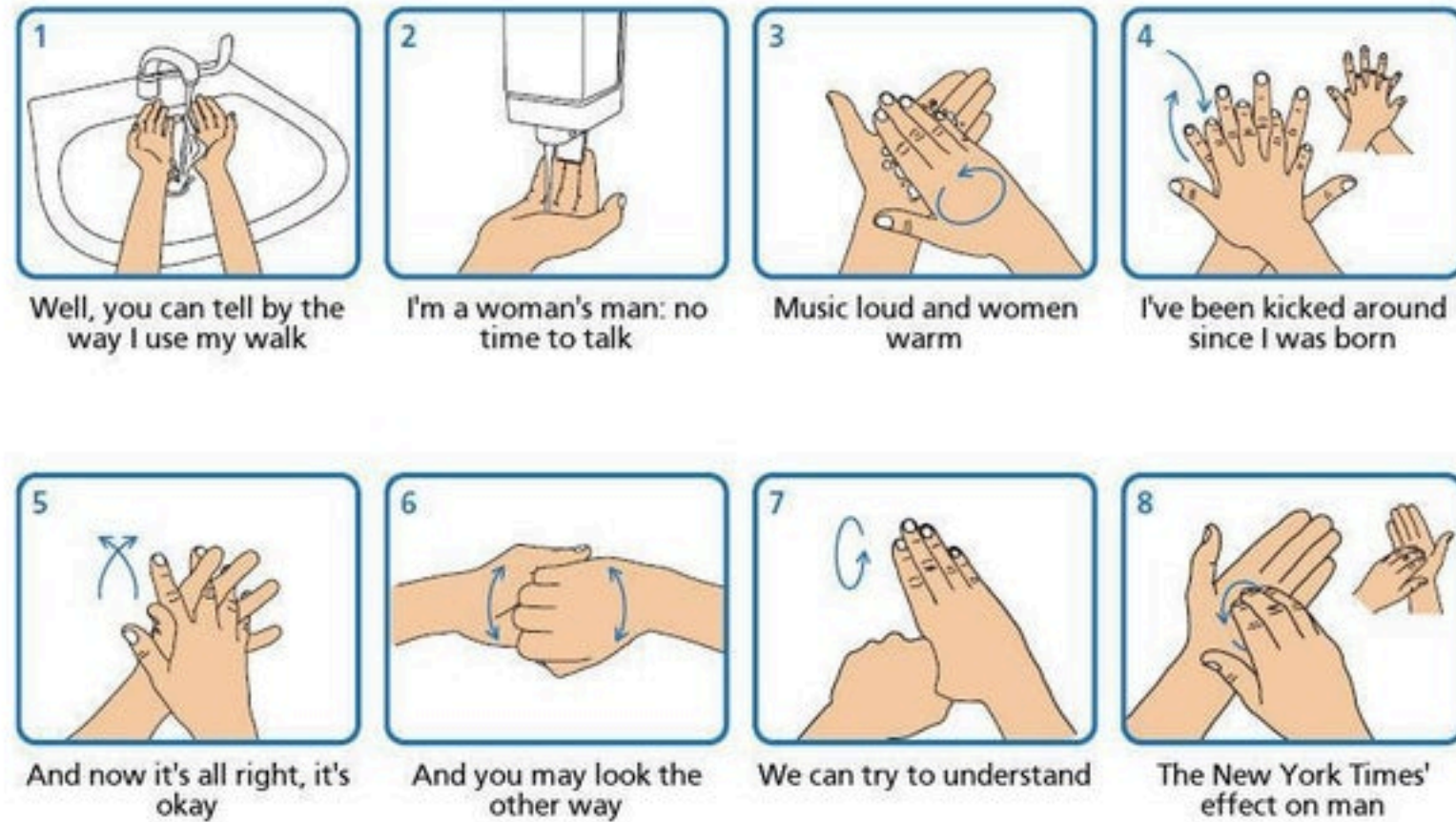
Modelling epidemic prevention measures

- **Reducing infection probability:** encouraging handwashing, cleaning surfaces, wearing masks
- **Removal of nodes from network:** quarantine, vaccination
- **Reduction of average node degree:** encouraging social distancing
- **Removal of edges between communities:** travel restriction

Epidemic prevention: Reducing “R0”

- Reducing the chance of transmission from person to person.
- Handwashing techniques, wearing a mask, keeping 1m+ apart

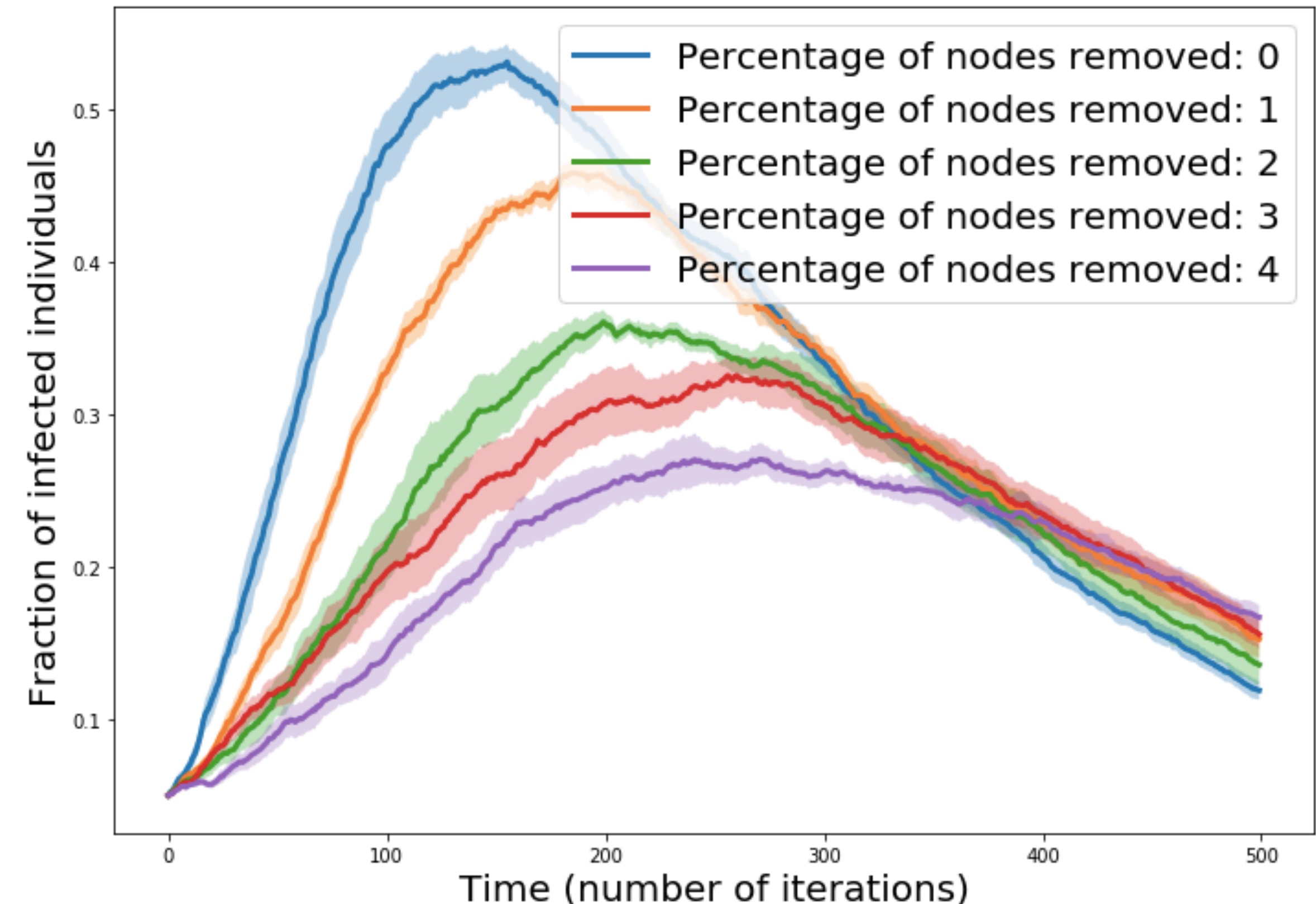
Hand-washing technique with soap and water



Reducing R0 not only reduces the **size of peak**, but pushes it **later in time**

Epidemic prevention: removing nodes/links

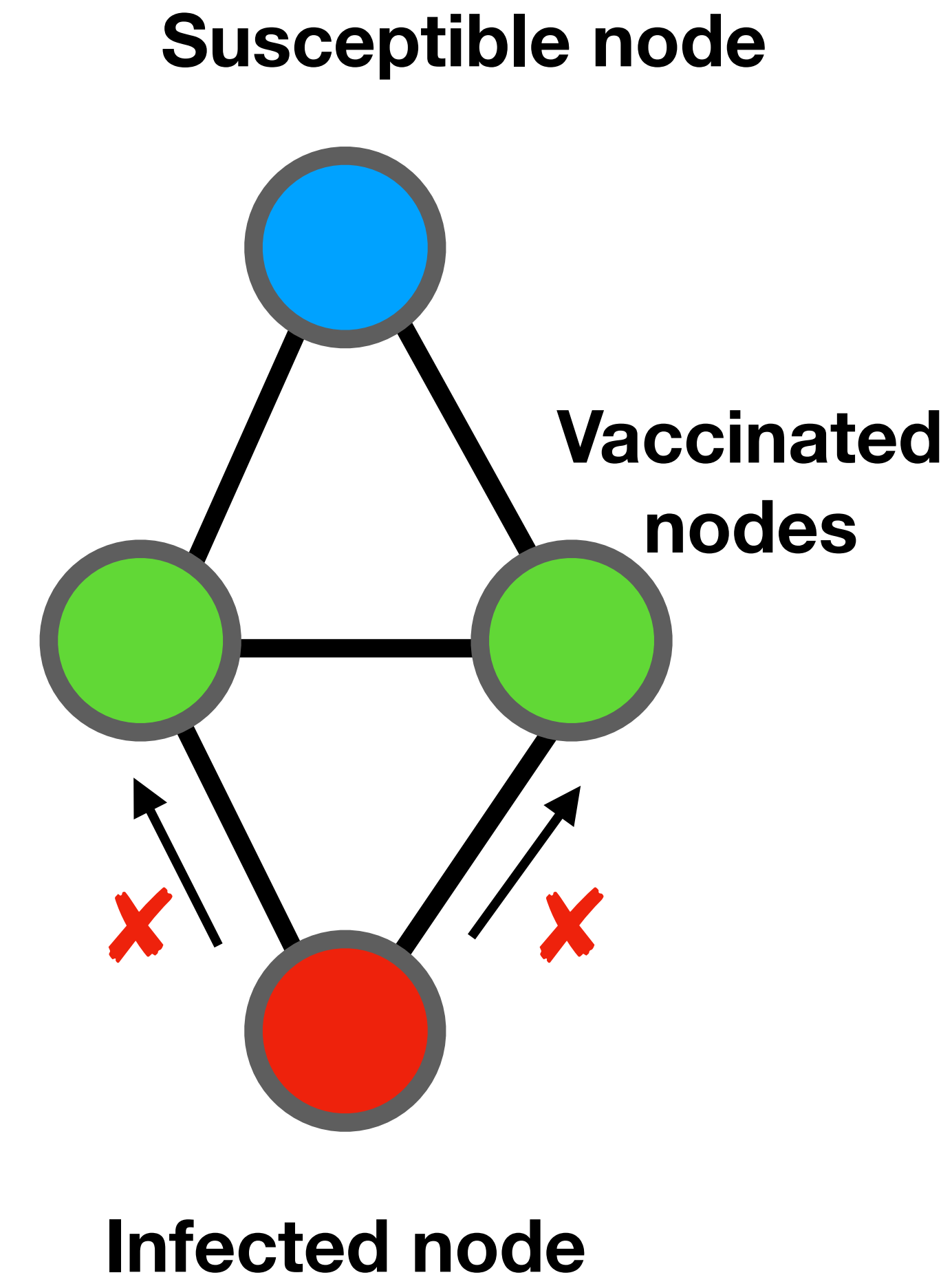
- Remove **nodes** — vaccination or quarantine of certain individuals
- Remove **edges** — pair of individuals cut contact altogether
- (in transport network, mobility restrictions)



Targeted immunisation in scale-free network (Pastor-Satorras et al) — removing nodes **reduces and pushes back** the infection peak, but gains decrease af

Herd immunity

- Cannot vaccinate **whole** of population (some too **vulnerable** to be vaccinated)
- Depending on network structure, if **enough people** are immune, the disease will die out (immune people act as blockers)



Conclusions

- **Models of epidemics** on networks can be key for providing insights into how a disease spreads through a population.
- **More complex** models available which can give more **precise guidance** on measures to suppress or mitigate epidemics.
- **Network structure** plays a huge role (scale-free vs random, modular structure)
- **Challenges:** the true “network” is often **unknowable** and spreading processes are complex, **economic and social consequences** to whichever course of action taken that are hard to predict.