Mixed and time-varying models for network formation

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4 July 2019

Queen Mary
University of London

How do networks grow?

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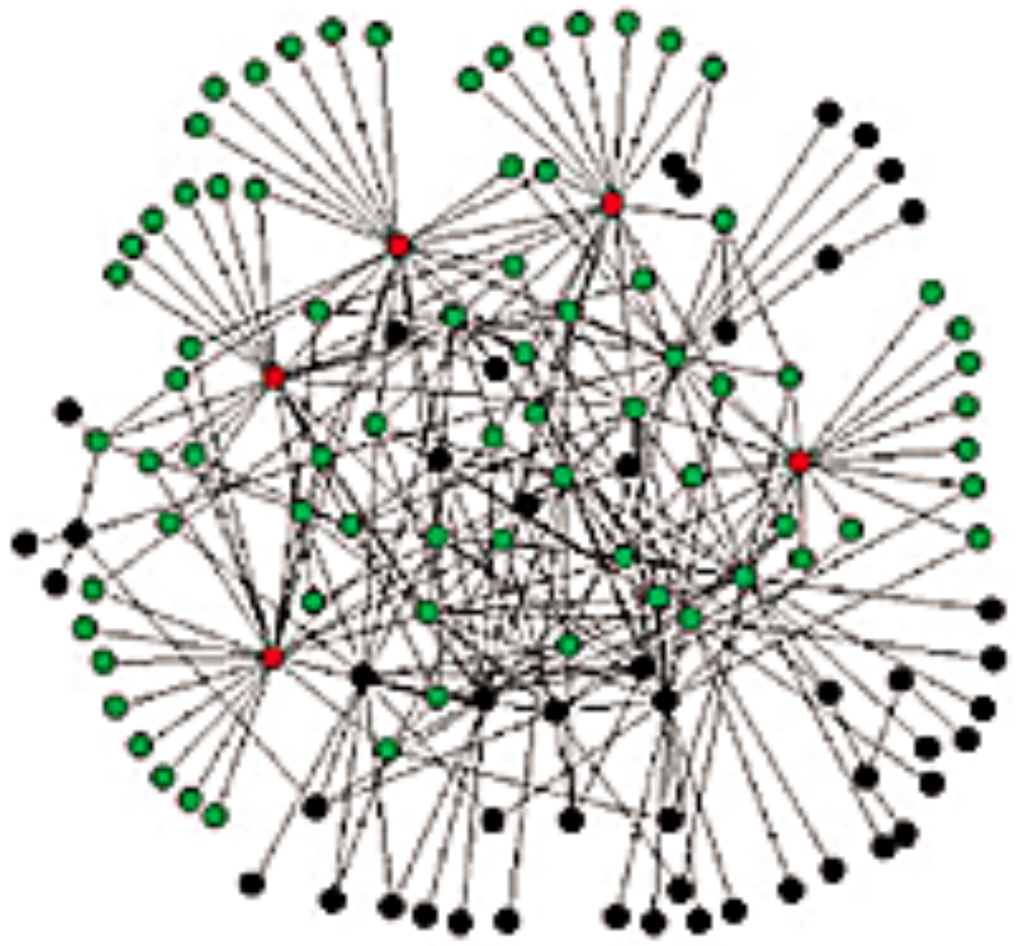
Emergence of Scaling in Random Networks

Albert-László Barabási* and Réka Albert

33759 Citations



Preferential Attachment

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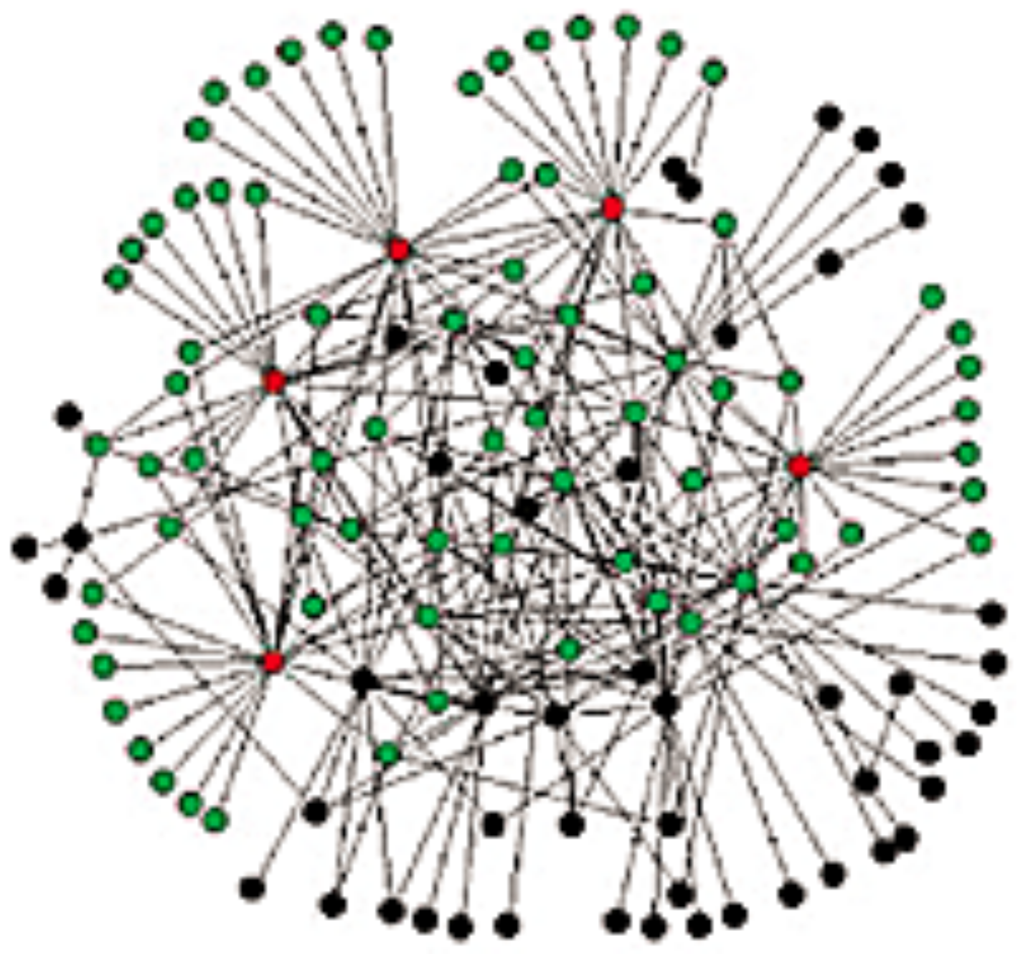

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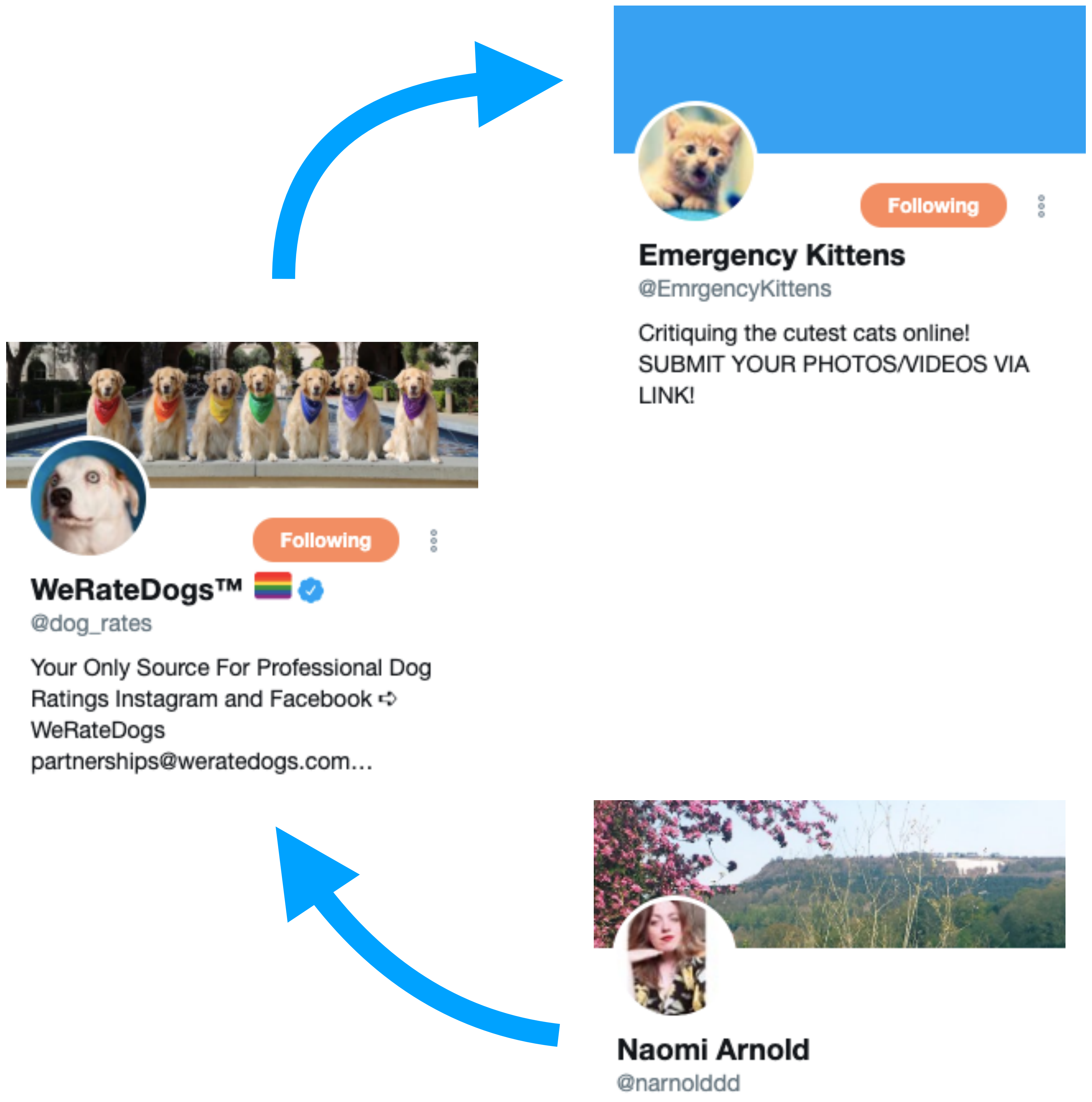

Triangle Closure

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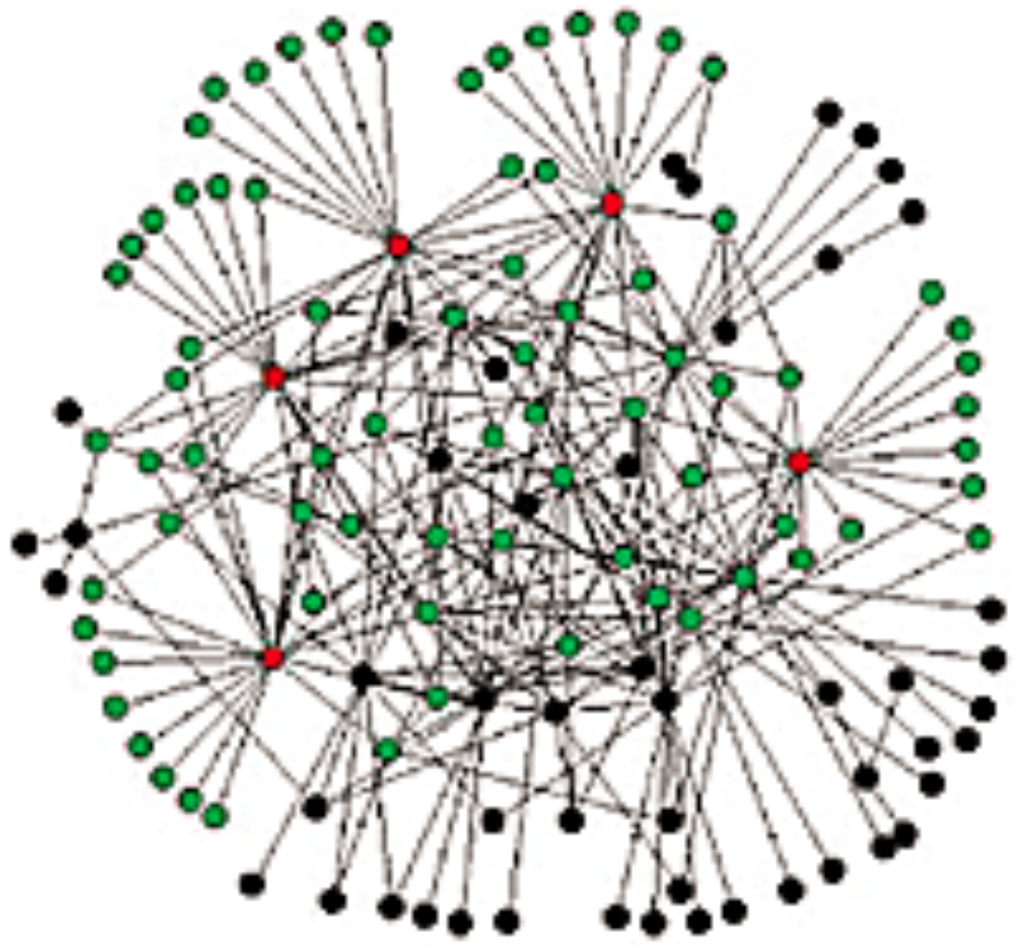

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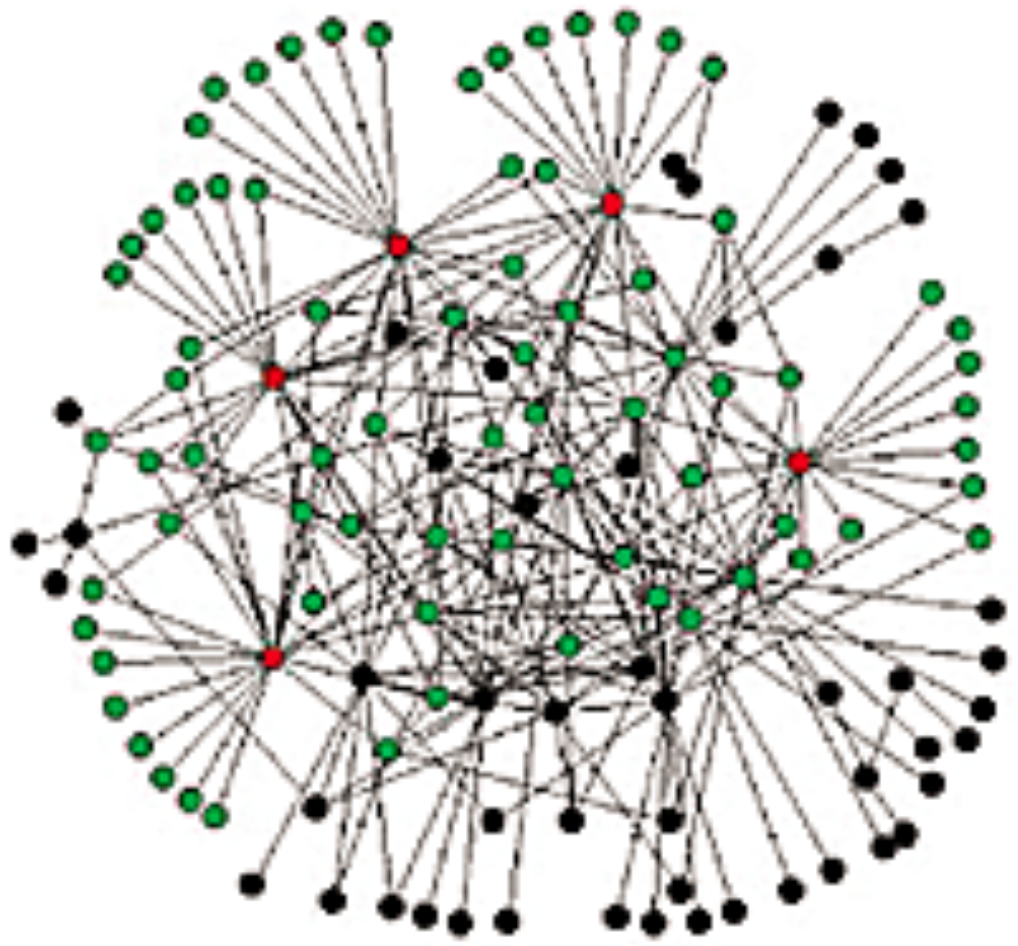


Preferential Attachment

Triangle Closure

Random Meeting

Our hypothesis

The model best describing growth of a network comprises a mixture of mechanisms…

… and this mixture may change over time.

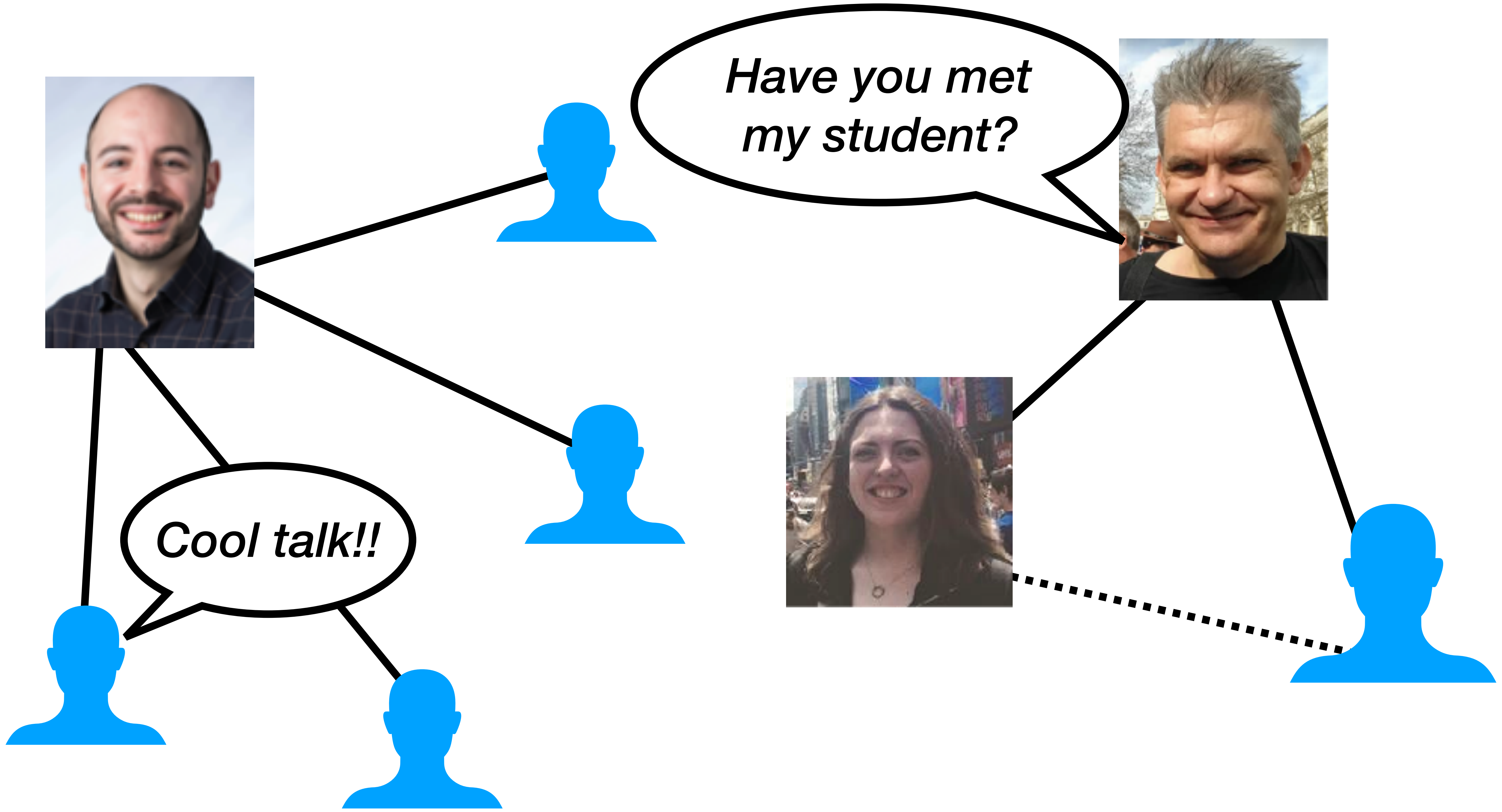
The Cosener's Example

Arrival coffee: Random interaction

Time

The Cosener's Example

Arrival coffee: Random interaction



Second Coffee session: Preferential Attachment/Triangle Closure

Time

The Cosener's Example

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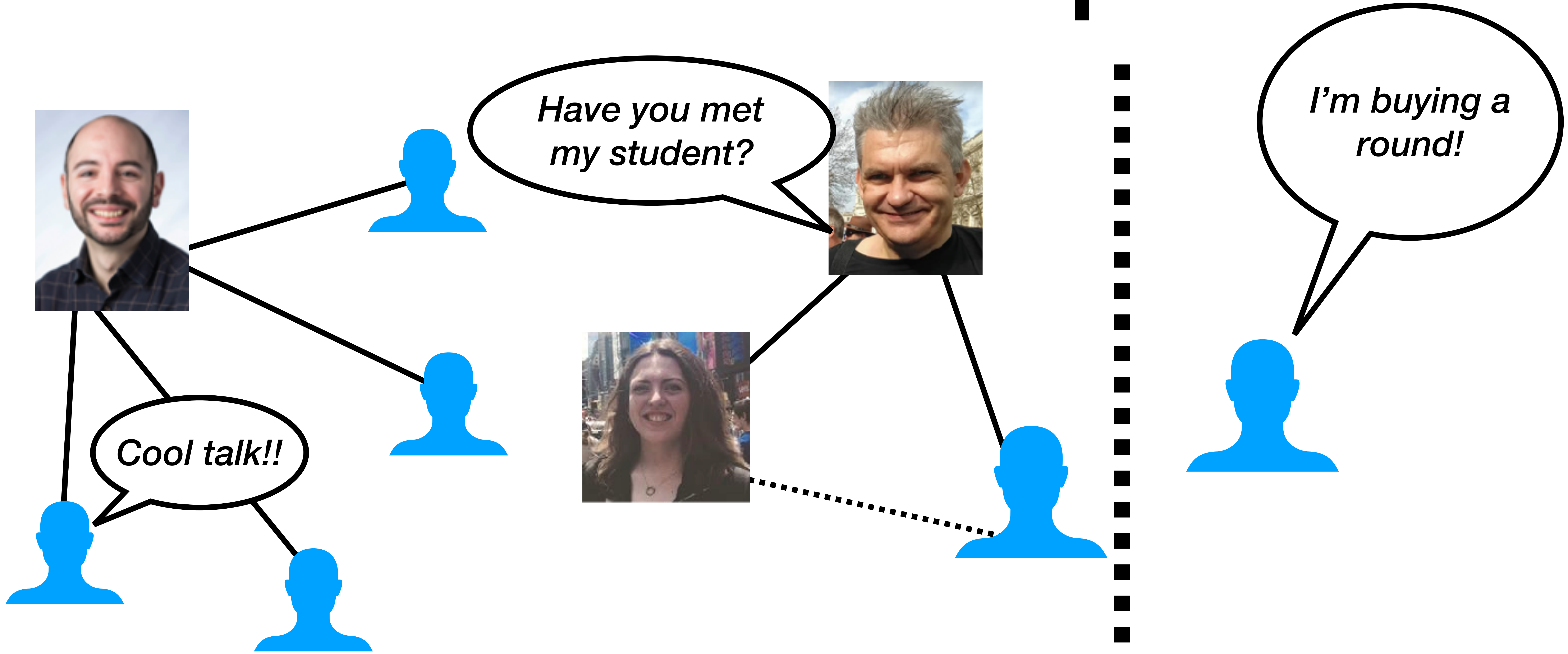

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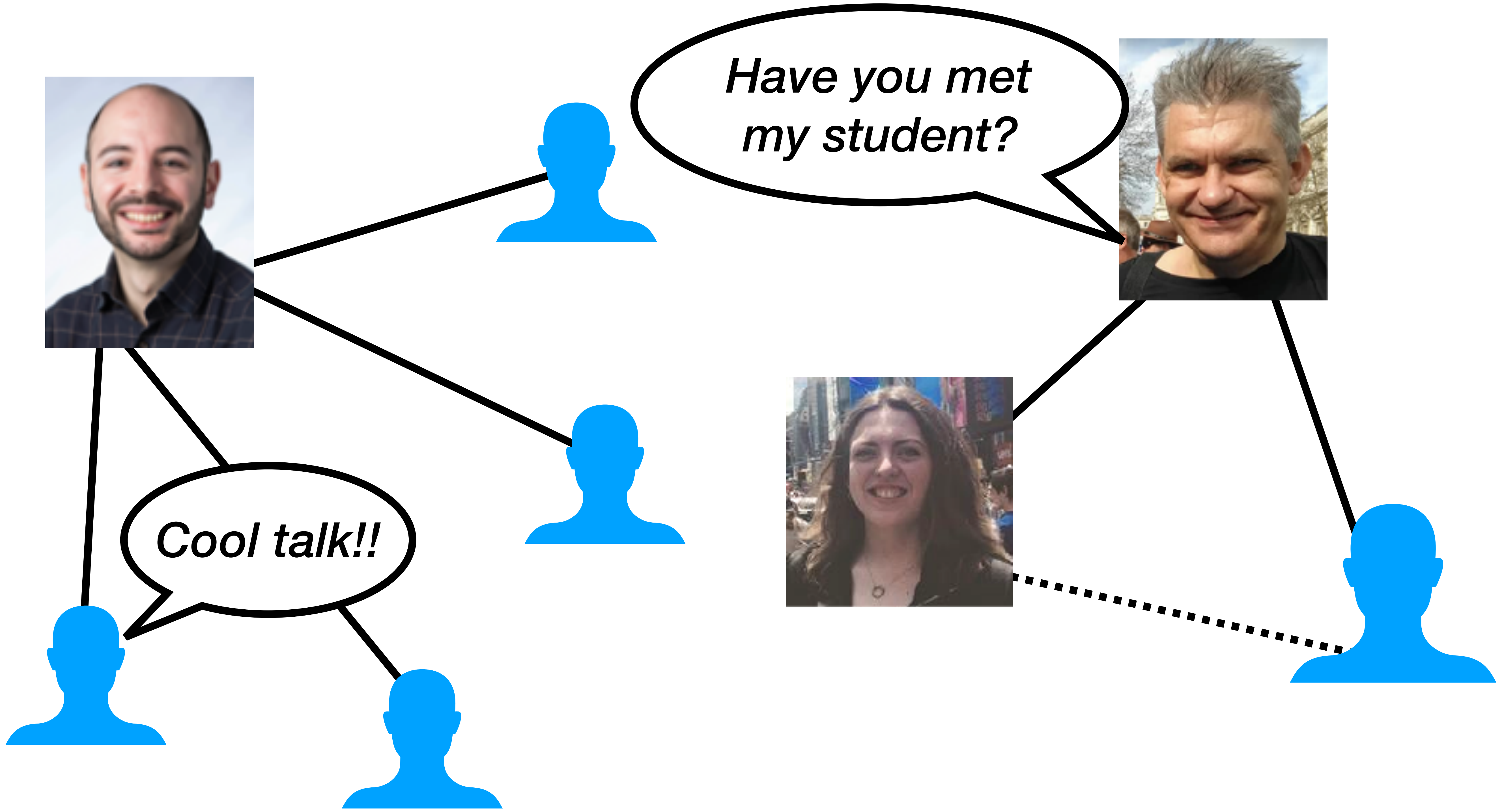

Second Coffee session: Preferential Attachment/Triangle Closure

Evening Social: "Other" preferential attachment

Time

Actual example: Enron email network

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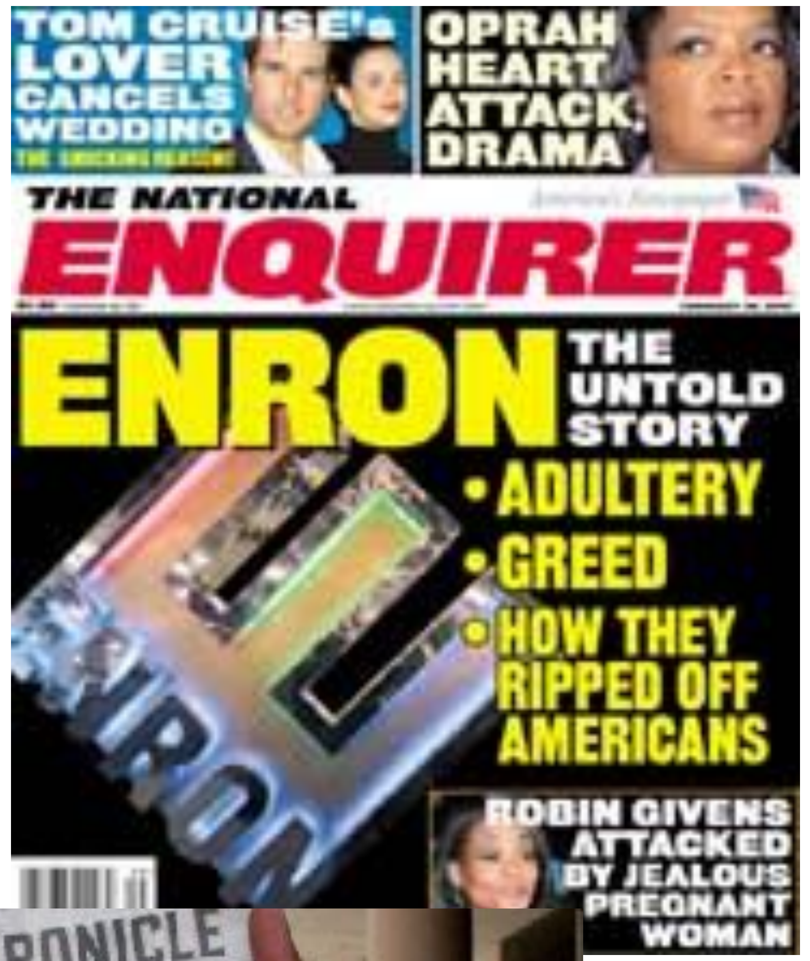

Enron scandal: multiple well-documented events in company's downfall

Actual example: Enron email network

Enron scandal: multiple well-documented events in company's downfall

3385	2935	1006874609
3385	2944	1006874609
3385	2951	1006874609
3385	2954	1006874609
3385	2956	1006874609
3385	2957	1006874609
3385	2975	1006874609
3385	2990	1006874609
3385	2991	1006874609
3385	2999	1006874609
3385	3024	1006874609
3385	3053	1006874609
3385	3387	1006874609

Sender Receiver UNIX

Corpus of emails between employees handed over for investigation

Actual example: Enron email network

Enron scandal: multiple well-documented events in company's downfall

?

Were events in the scandal reflected in the evolution of the email network?

```
3385 2935 1006874609
3385 2944 1006874609
3385 2951 1006874609
3385 2954 1006874609
3385 2956 1006874609
3385 2957 1006874609
3385 2975 1006874609
3385 2990 1006874609
3385 2991 1006874609
3385 2999 1006874609
3385 3024 1006874609
3385 3053 1006874609
3385 3387 1006874609
```

Sender

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UNIX

Corpus of emails between employees handed over for investigation

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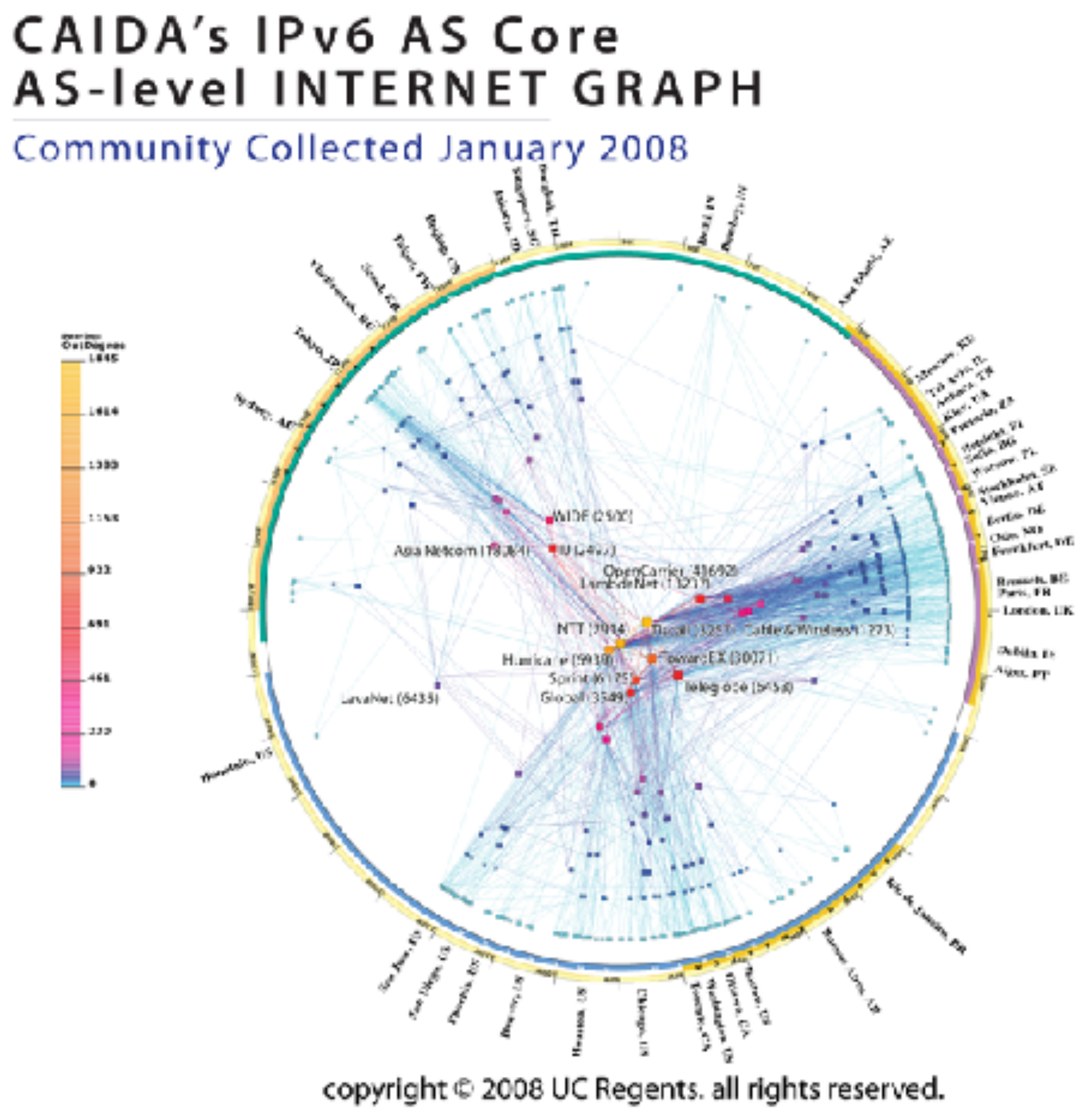

Network we want to model

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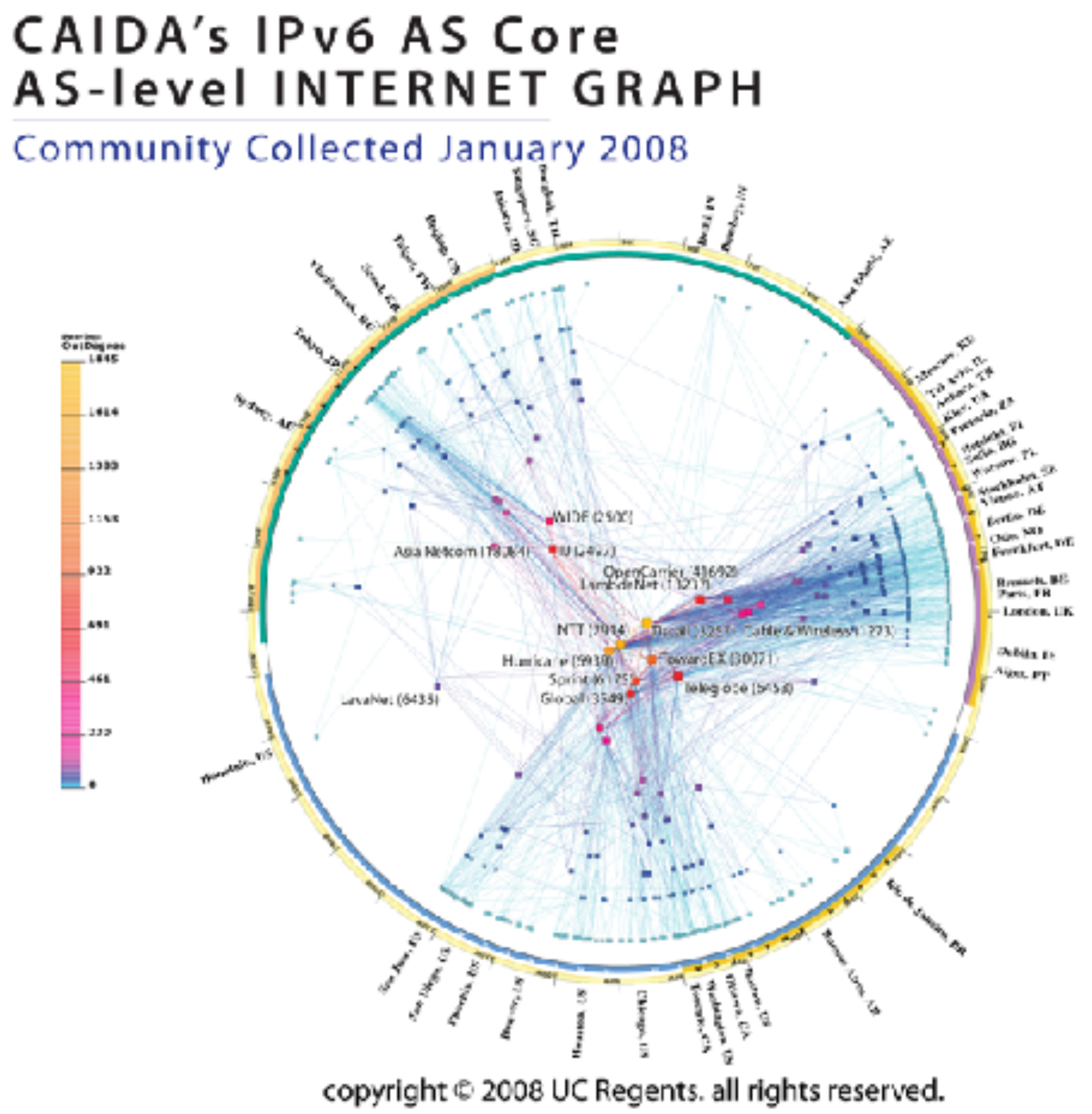

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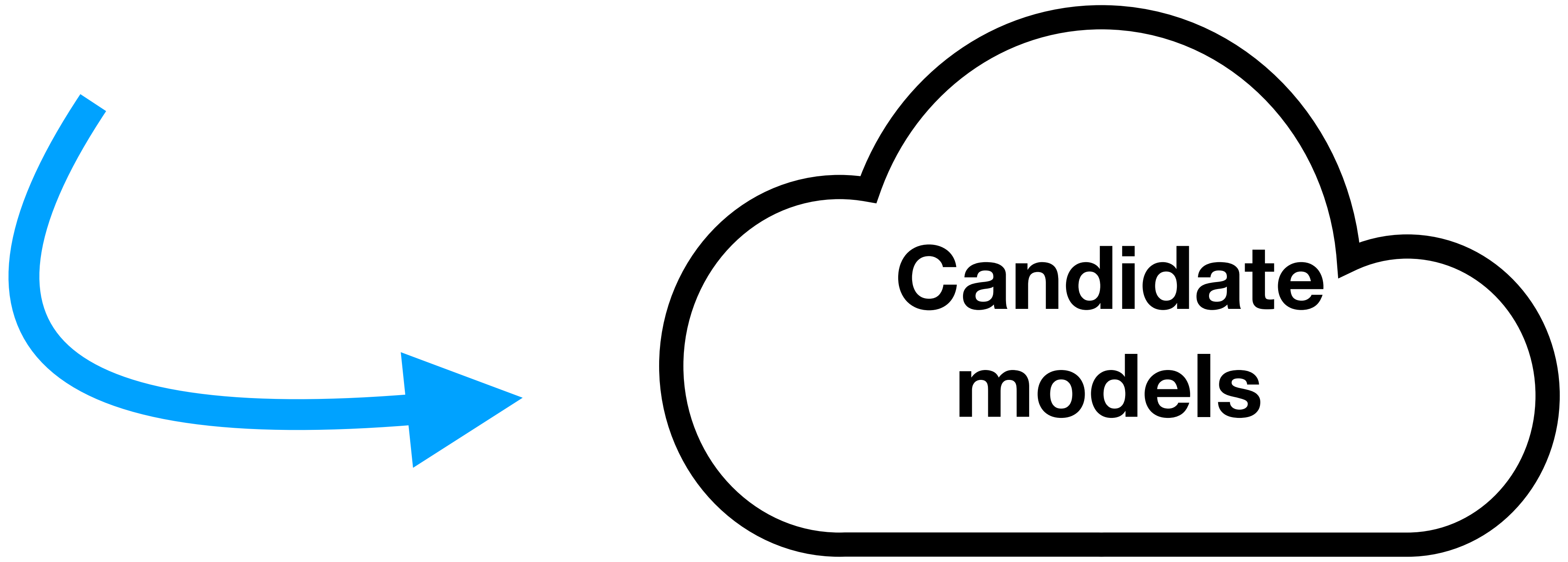

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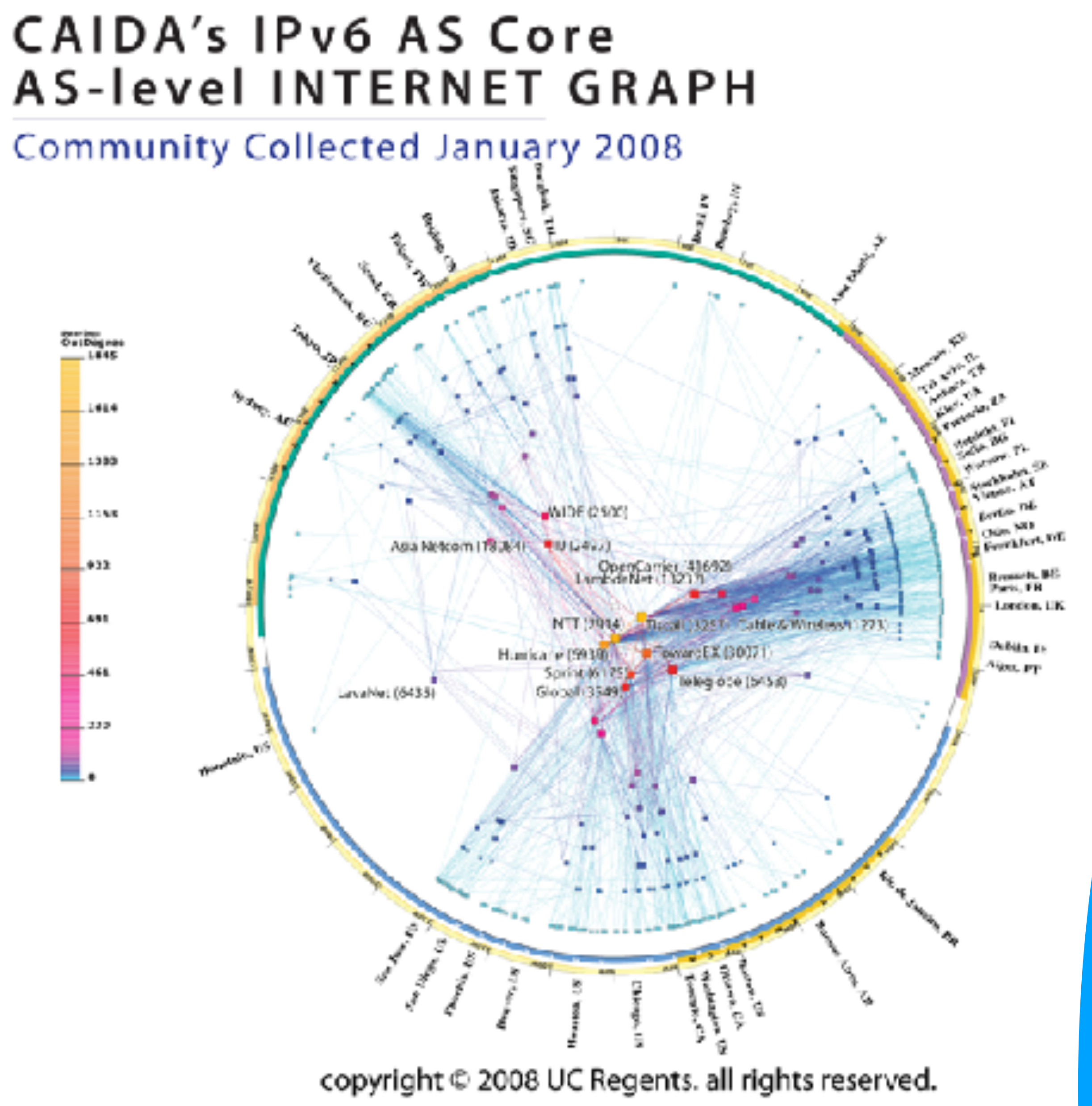

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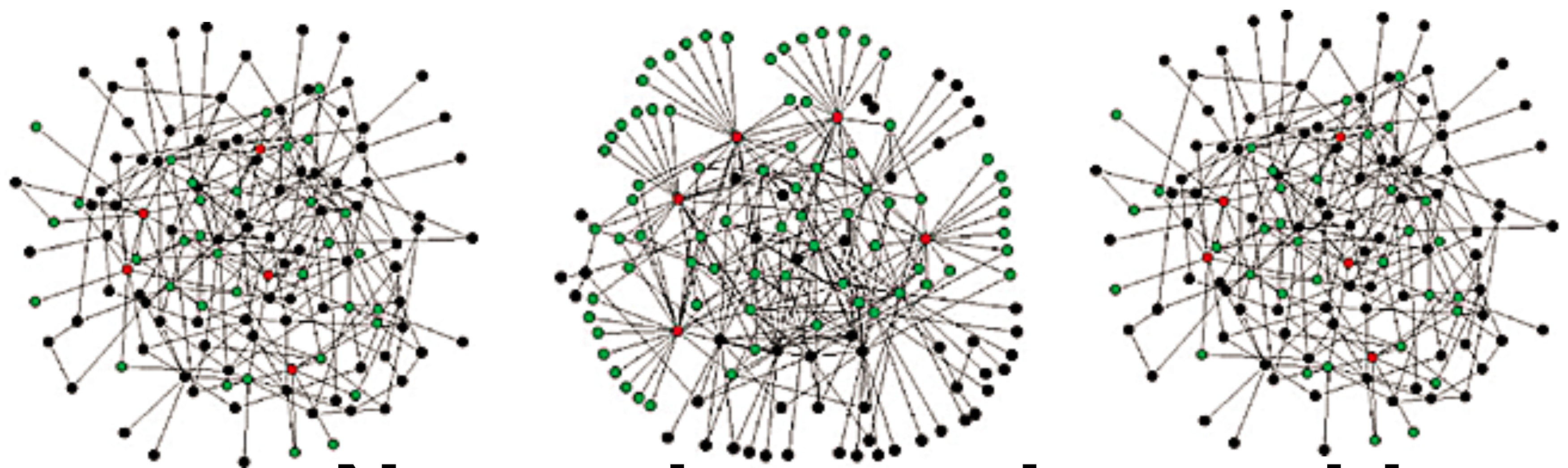

Network snapshots with same #nodes/links

Monte Carlo Simulations

Candidate models

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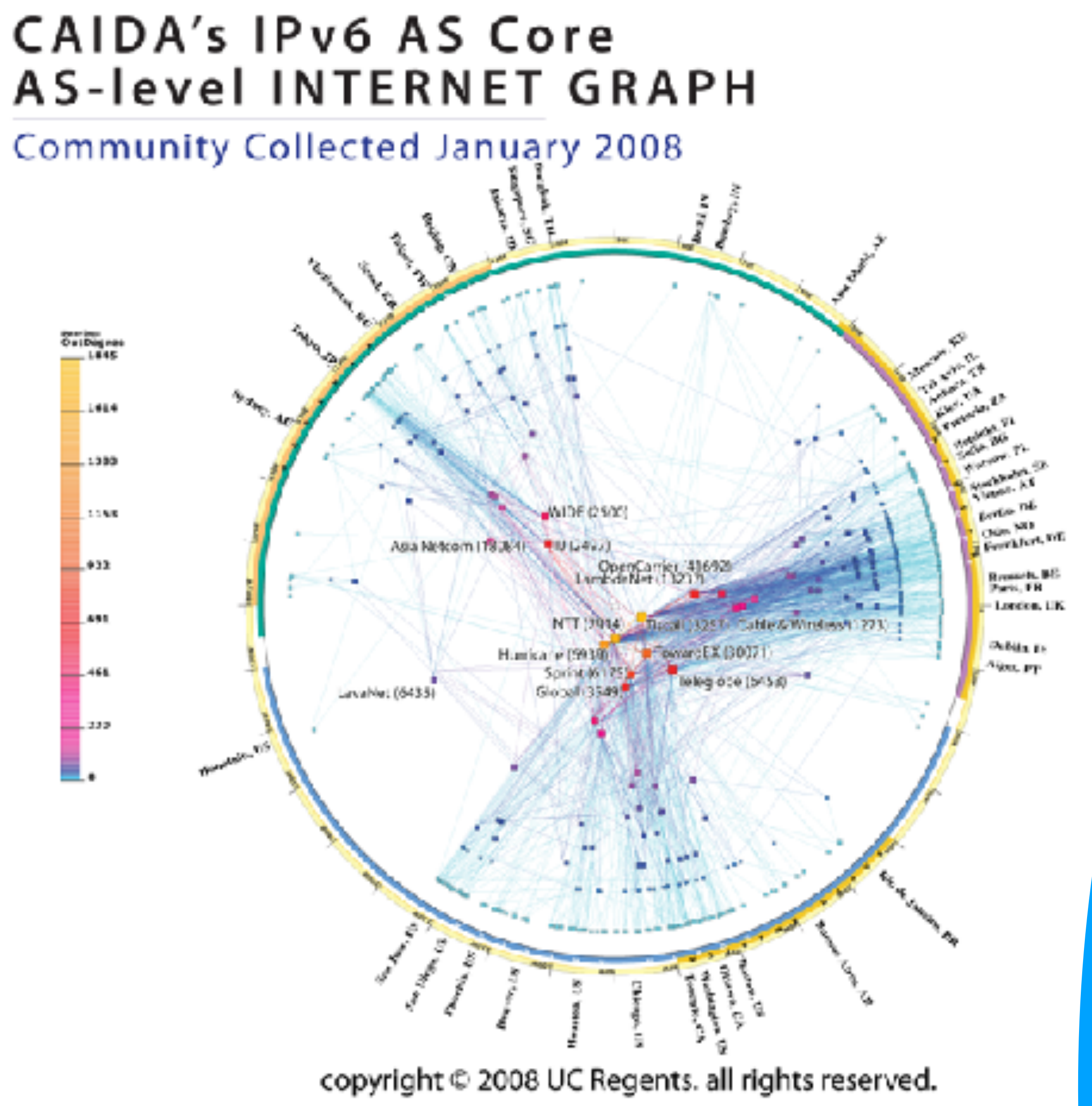

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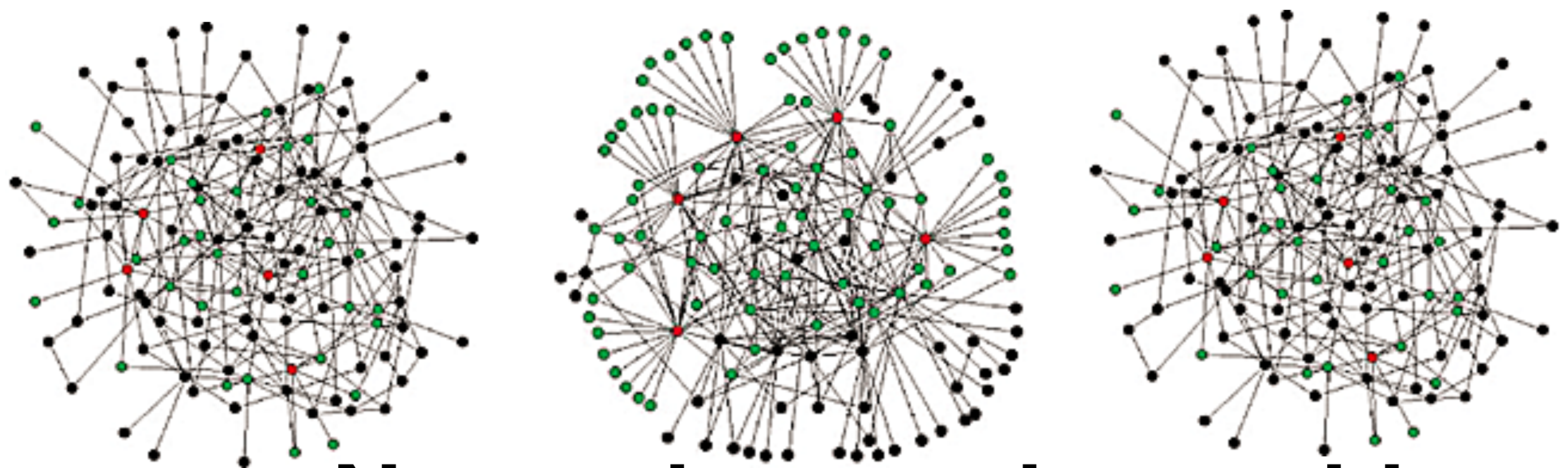
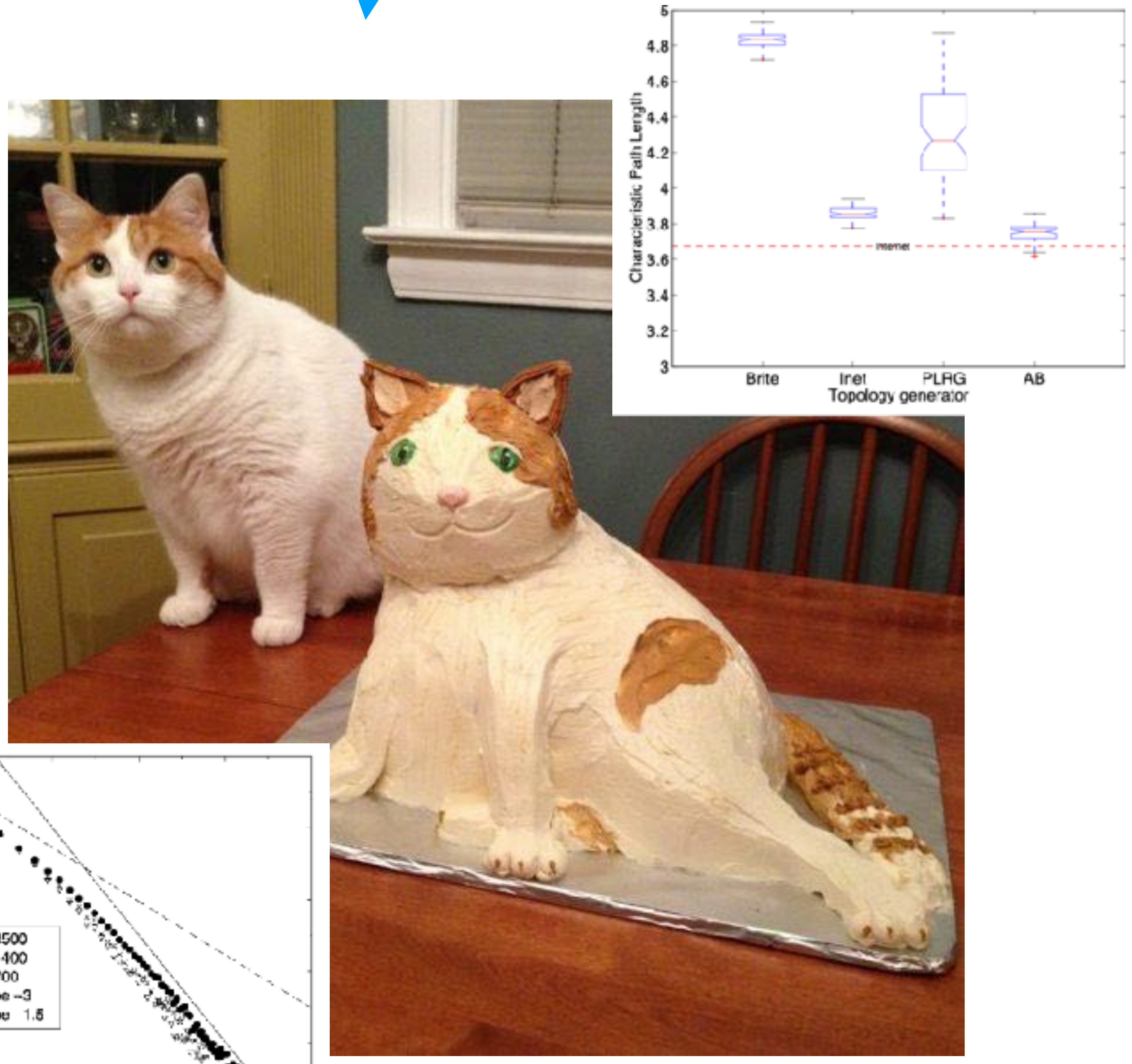


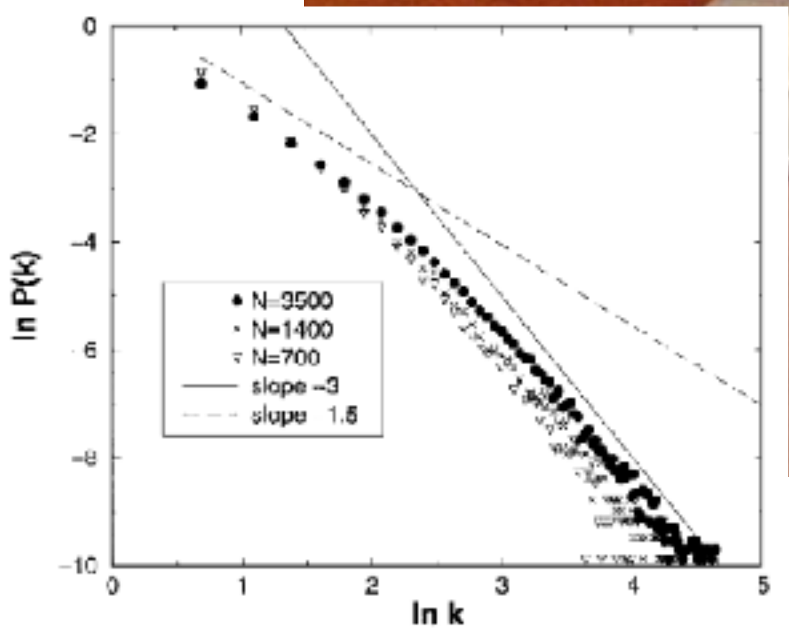
Network snapshots with same #nodes/links

Monte Carlo Simulations

Candidate models

Comparison of stats with original network





Shortfalls with this approach

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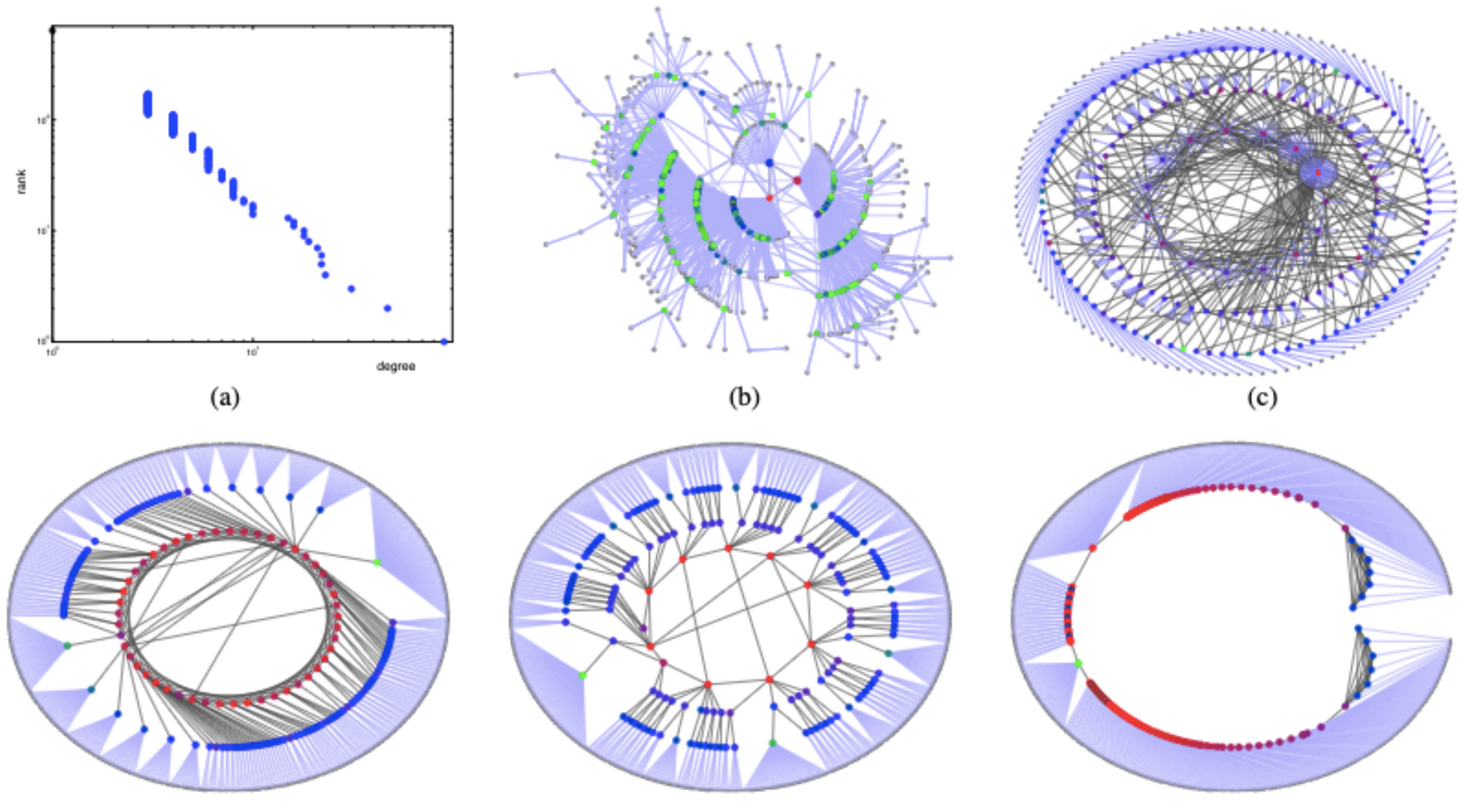

(a) (b) (c)

(d) (e) (f)

1. Networks can have same statistics (e.g. degree distribution) but dramatically different properties

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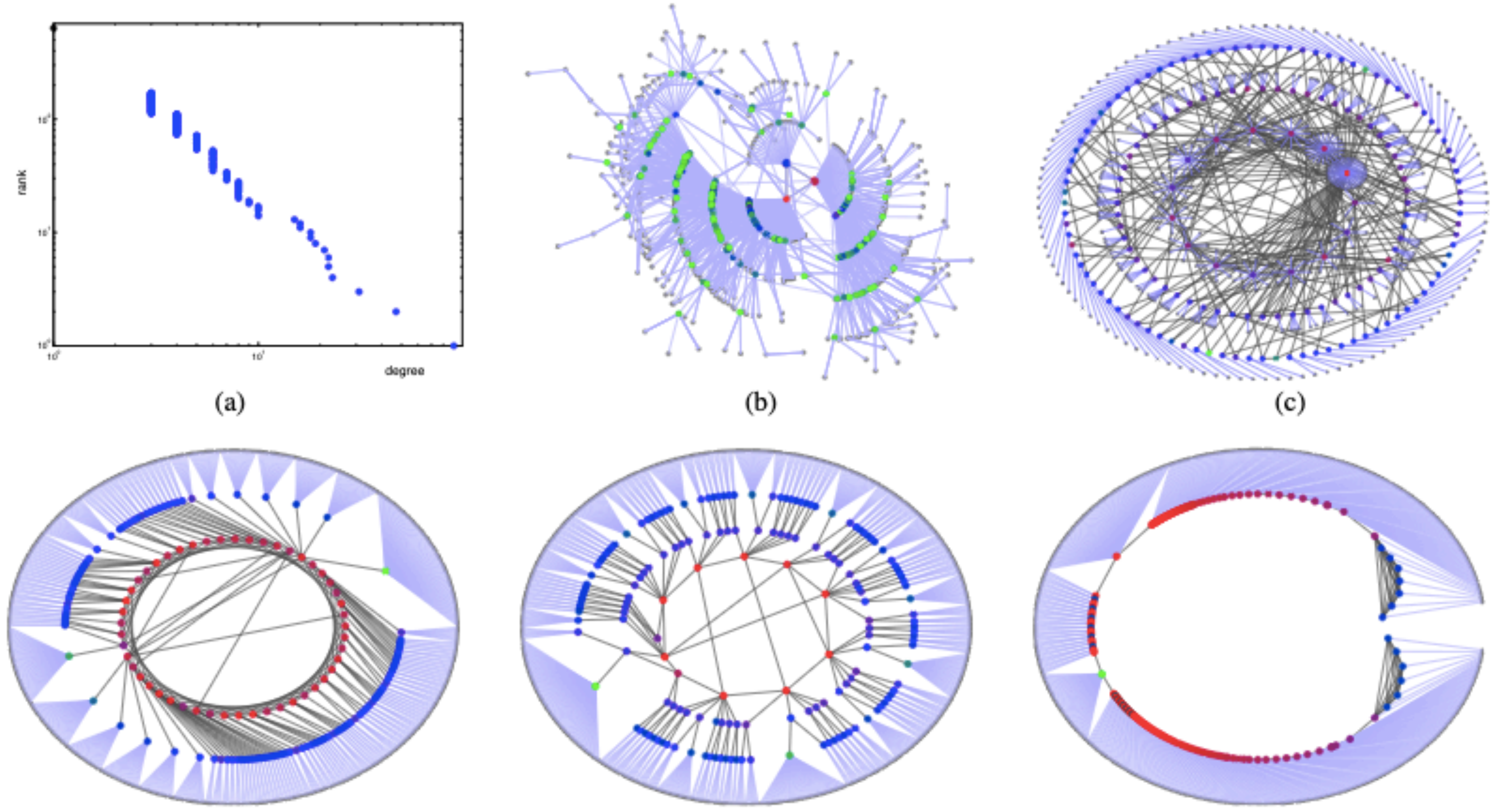

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2. Different models may perform better on different statistics

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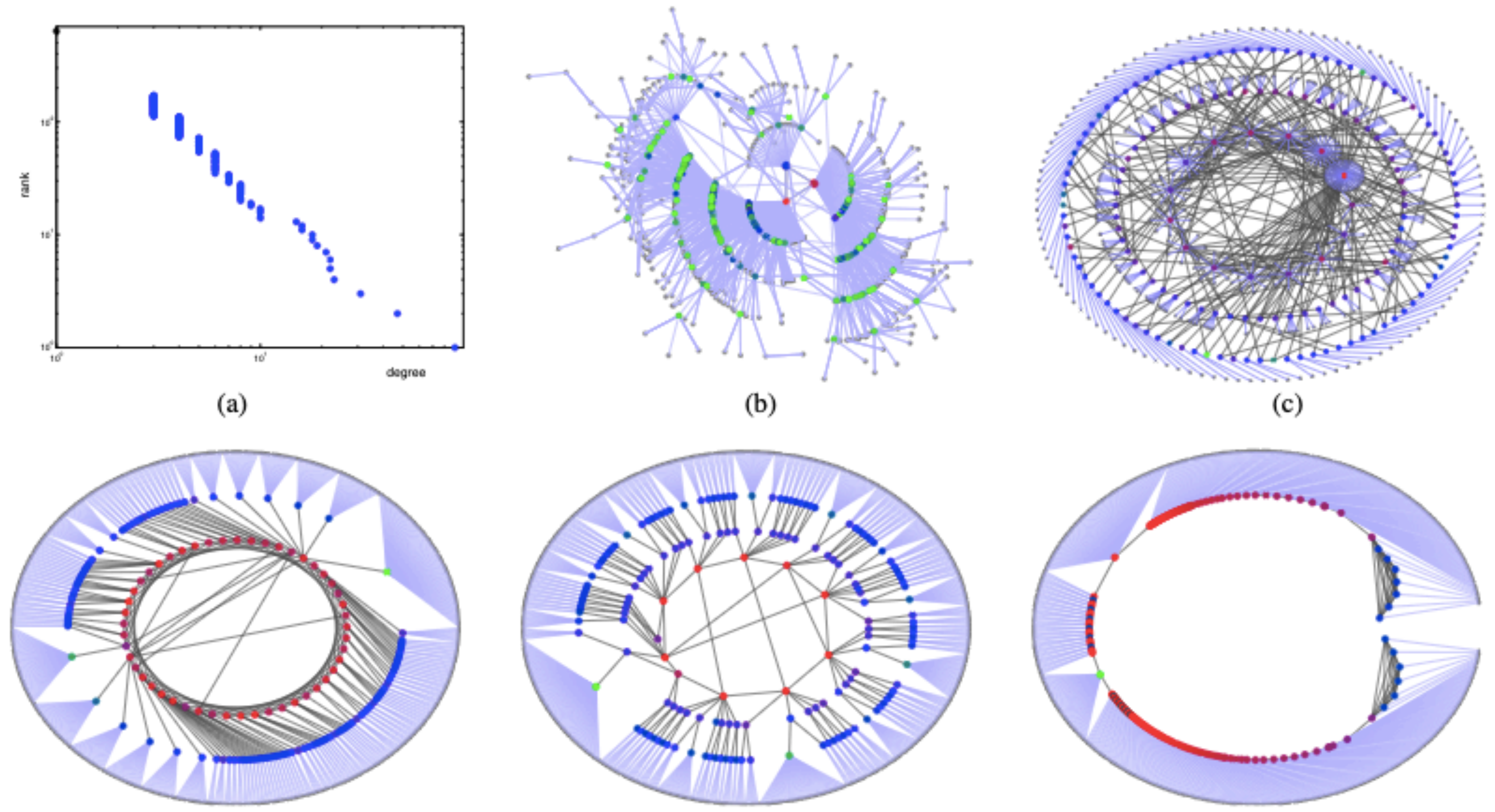

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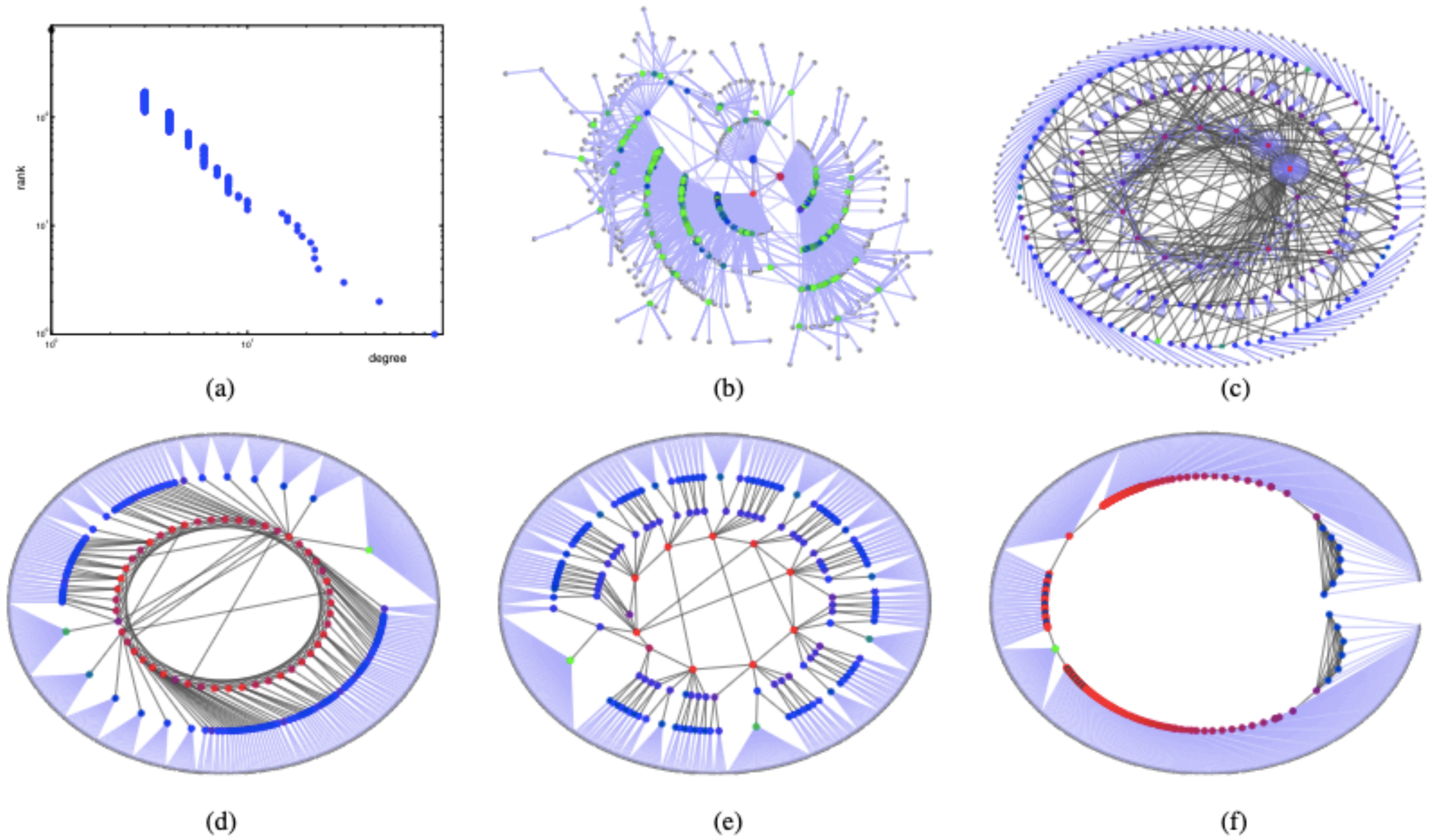

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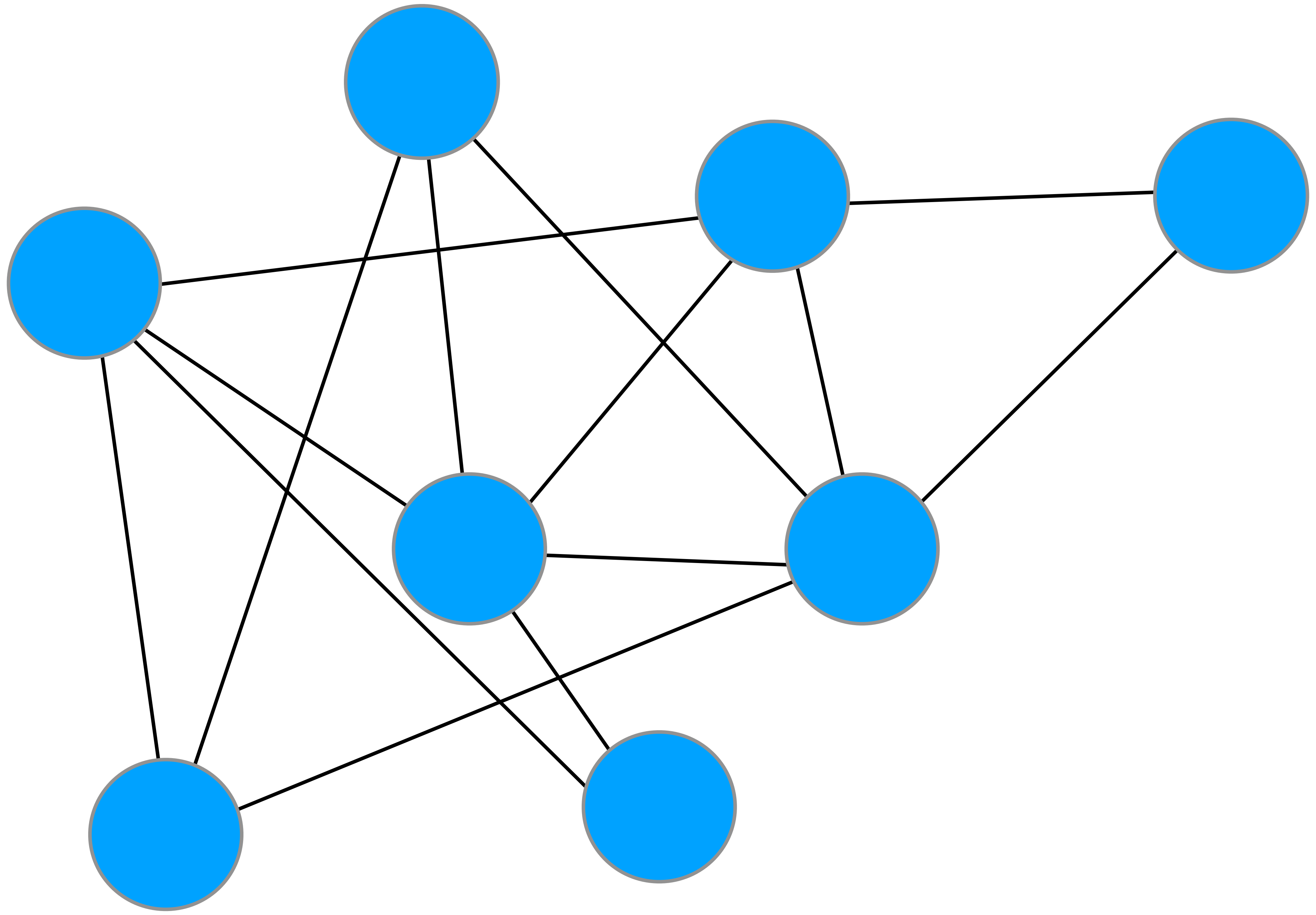
Hmm… What about with more information than just a snapshot?

With temporal information of link arrival times, we can do better!



vs

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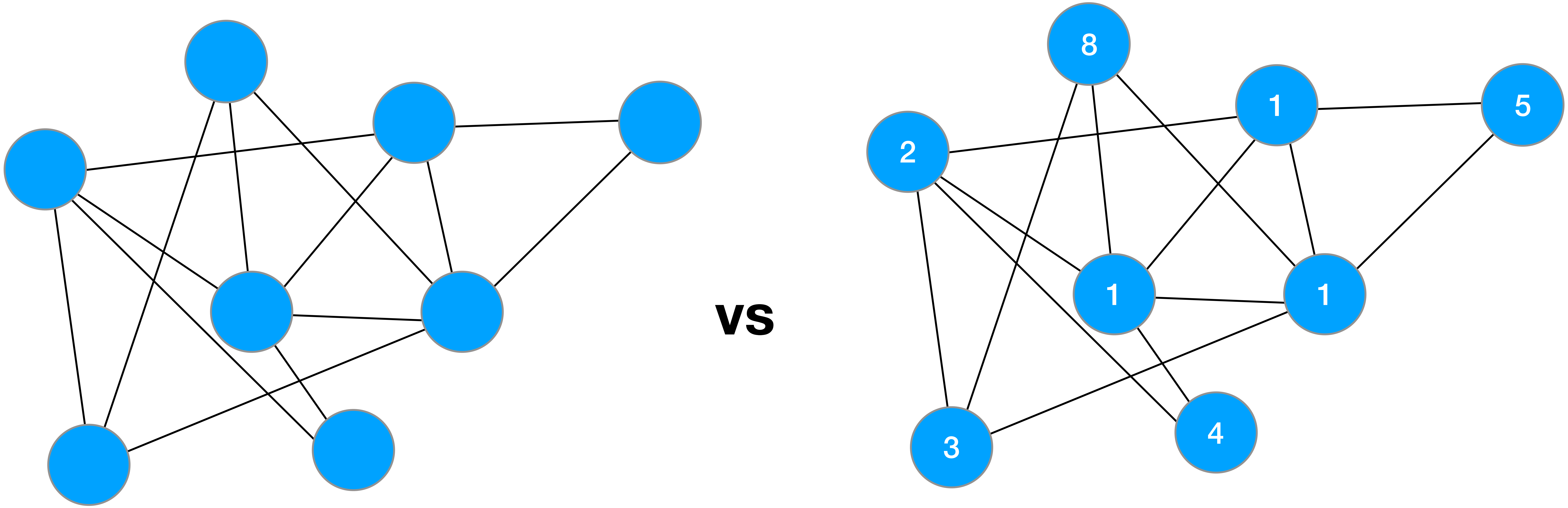

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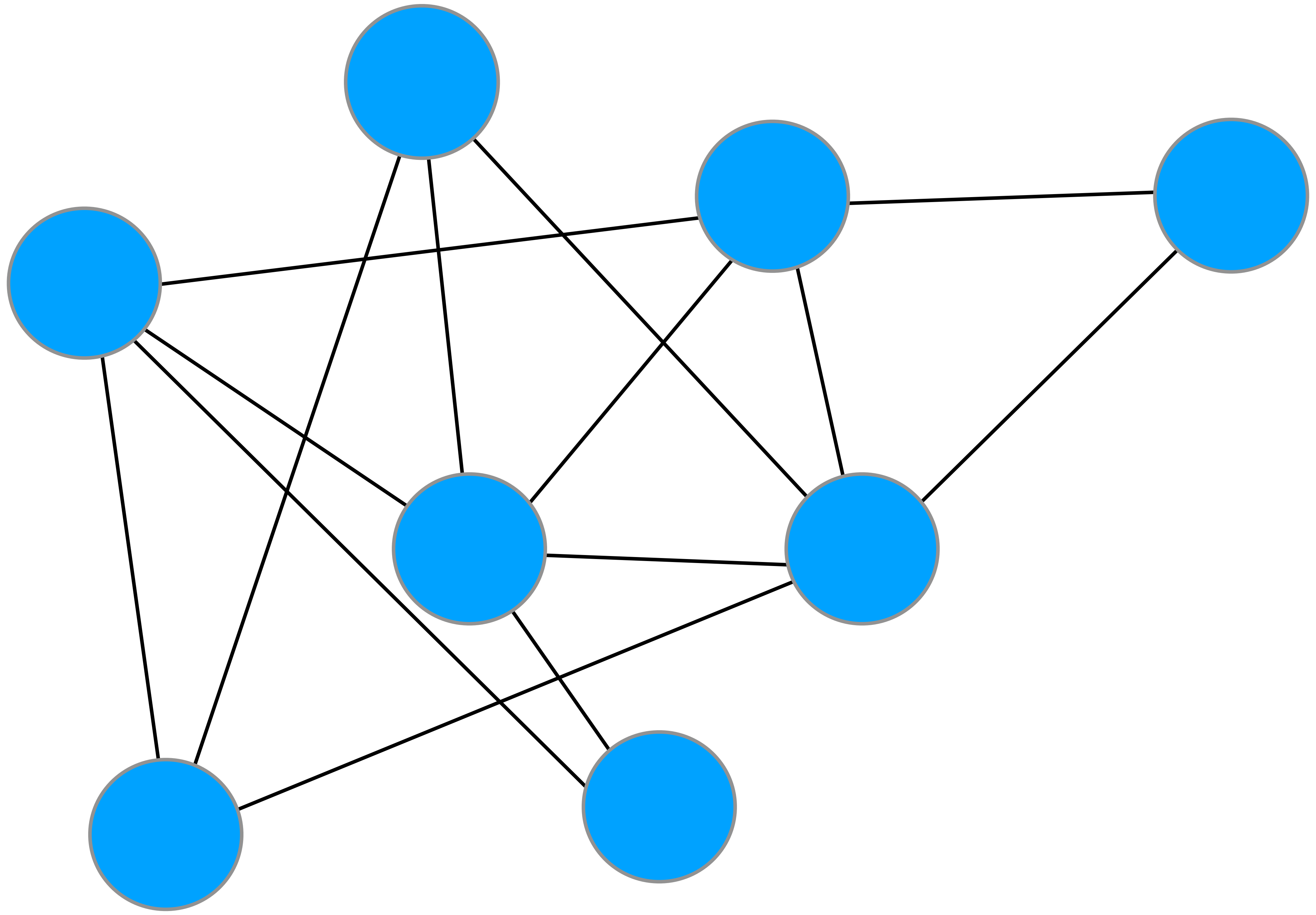

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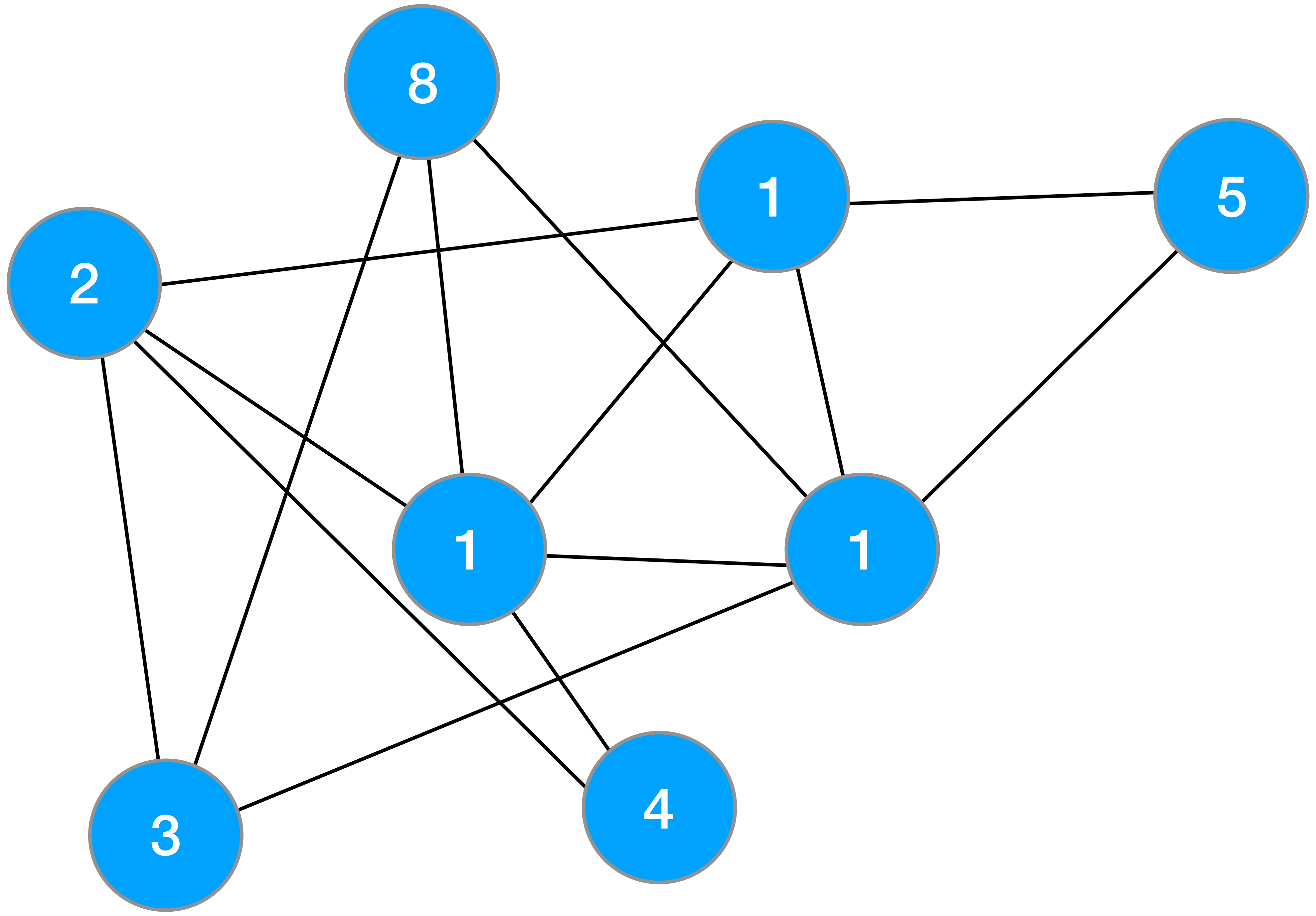

n=1 graph observations

n=8 observations of graph growth

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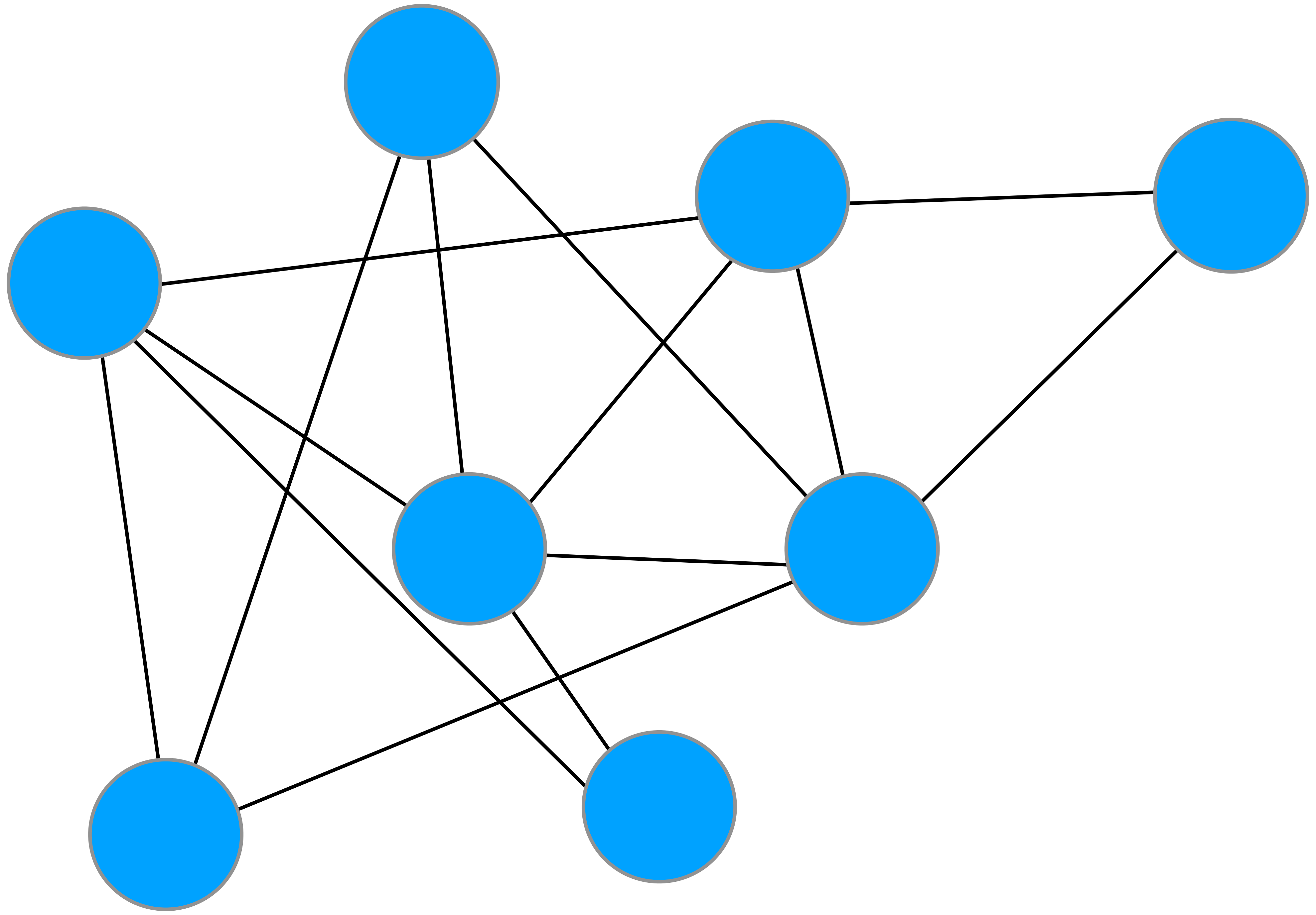

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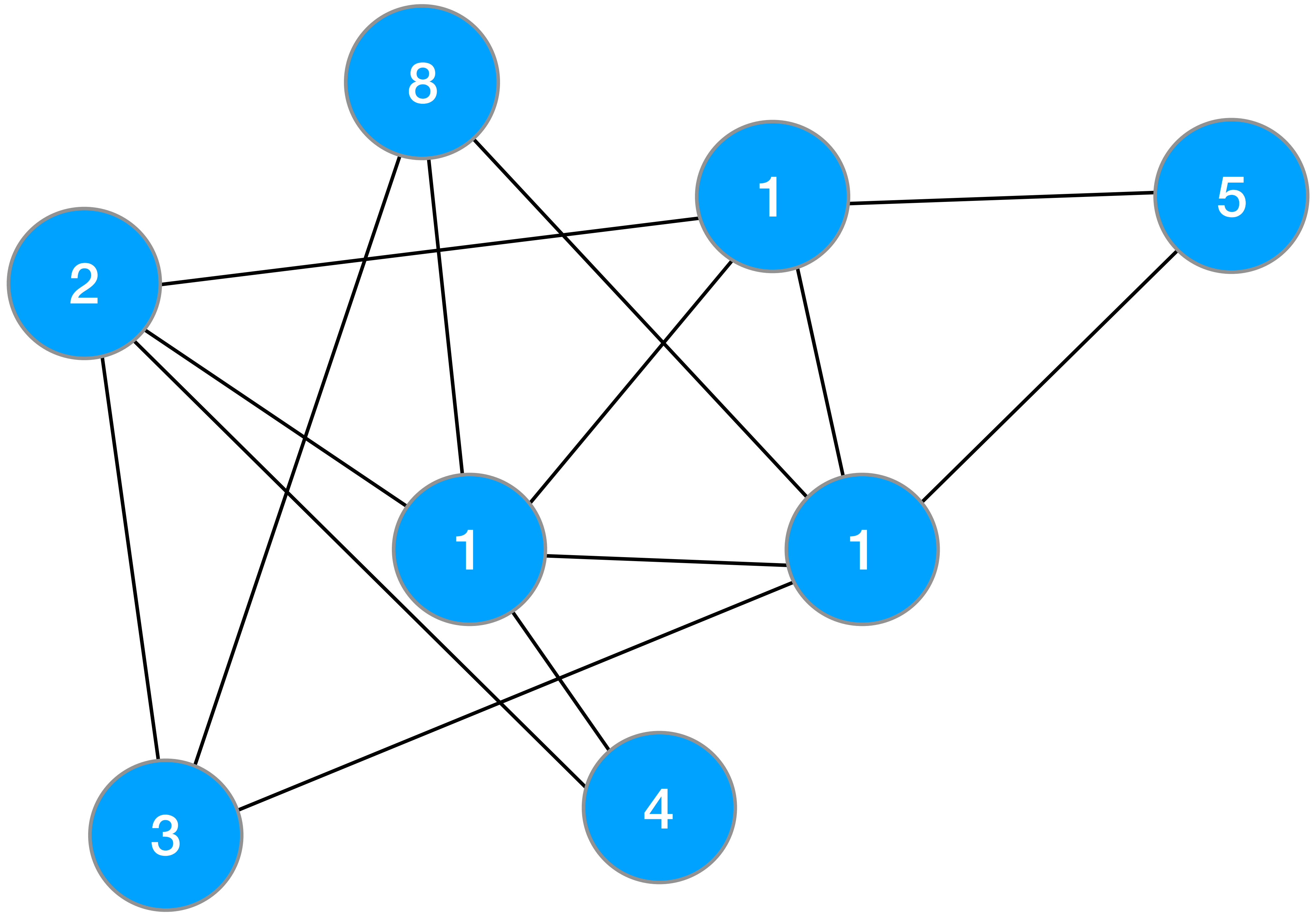
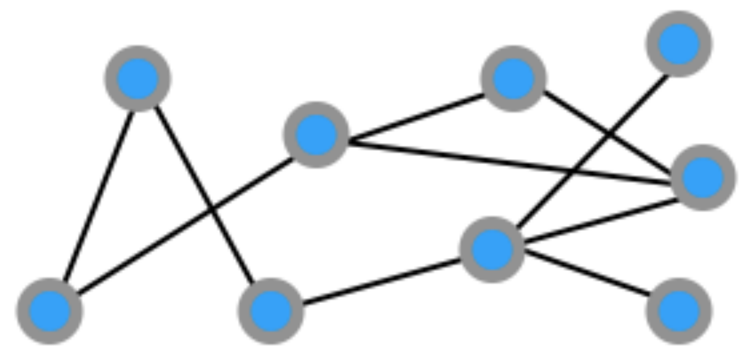


n=8 observations of graph growth

=> Can calculate precise likelihood of model, see R.Clegg et al: Likelihood based assessment of dynamic networks (2015)

What does such a model look like?

For any new node joining the network, or existing node choosing to make new connections, node i is chosen as a neighbour with probability:



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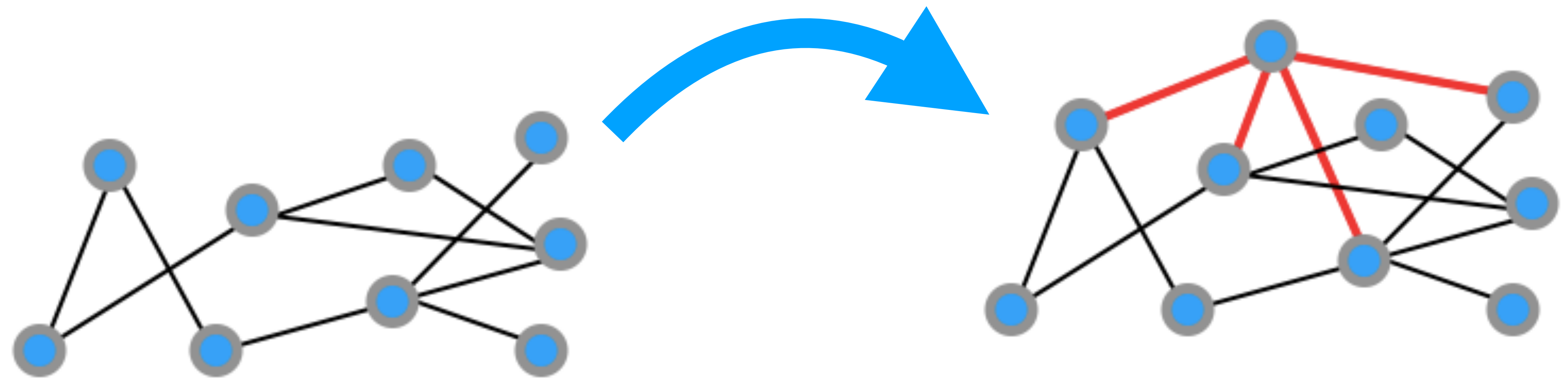

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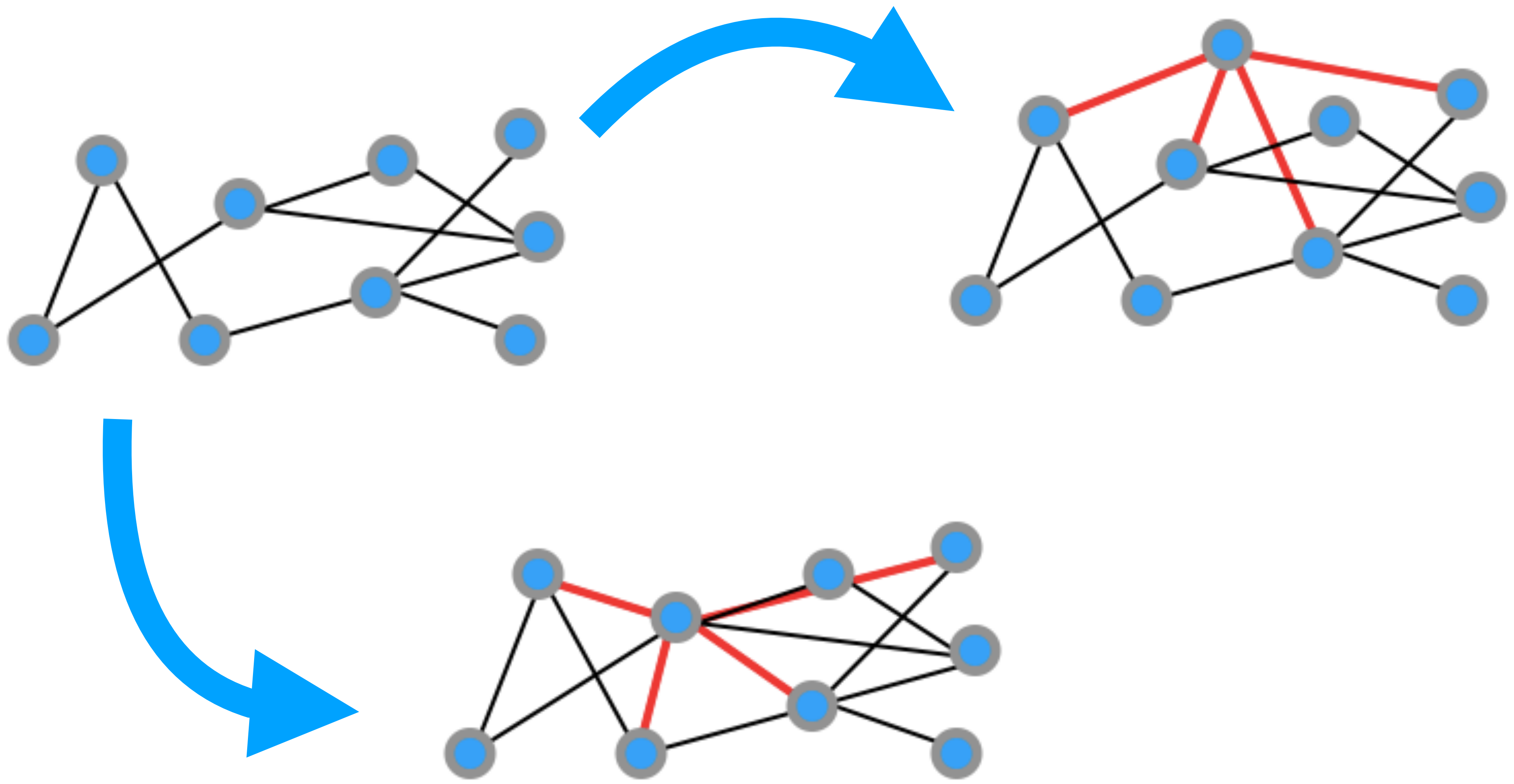

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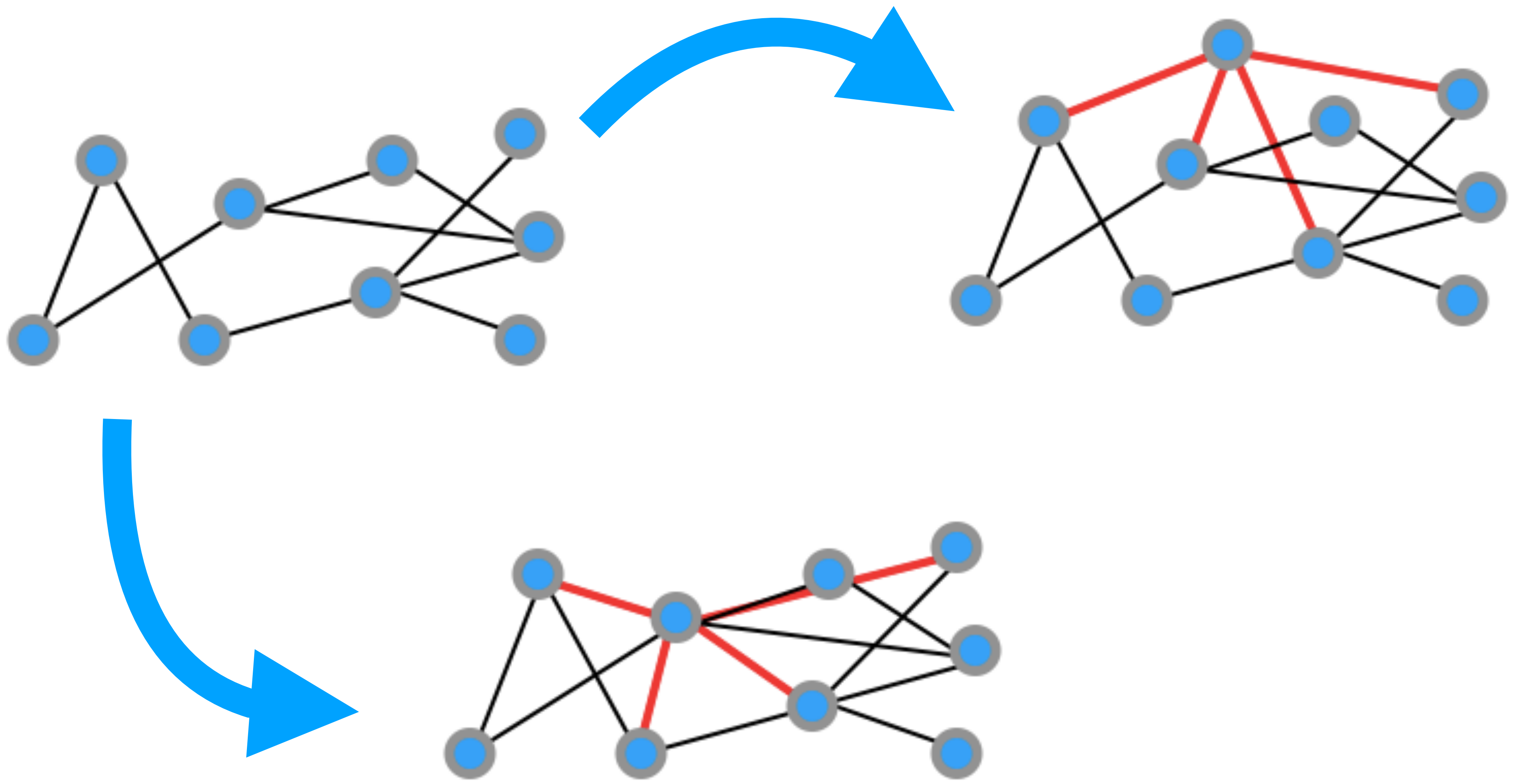

$$p_i = \beta_1(t)p_i^{(1)} + \ldots + \beta_l(t)p_i^{(l)}$$

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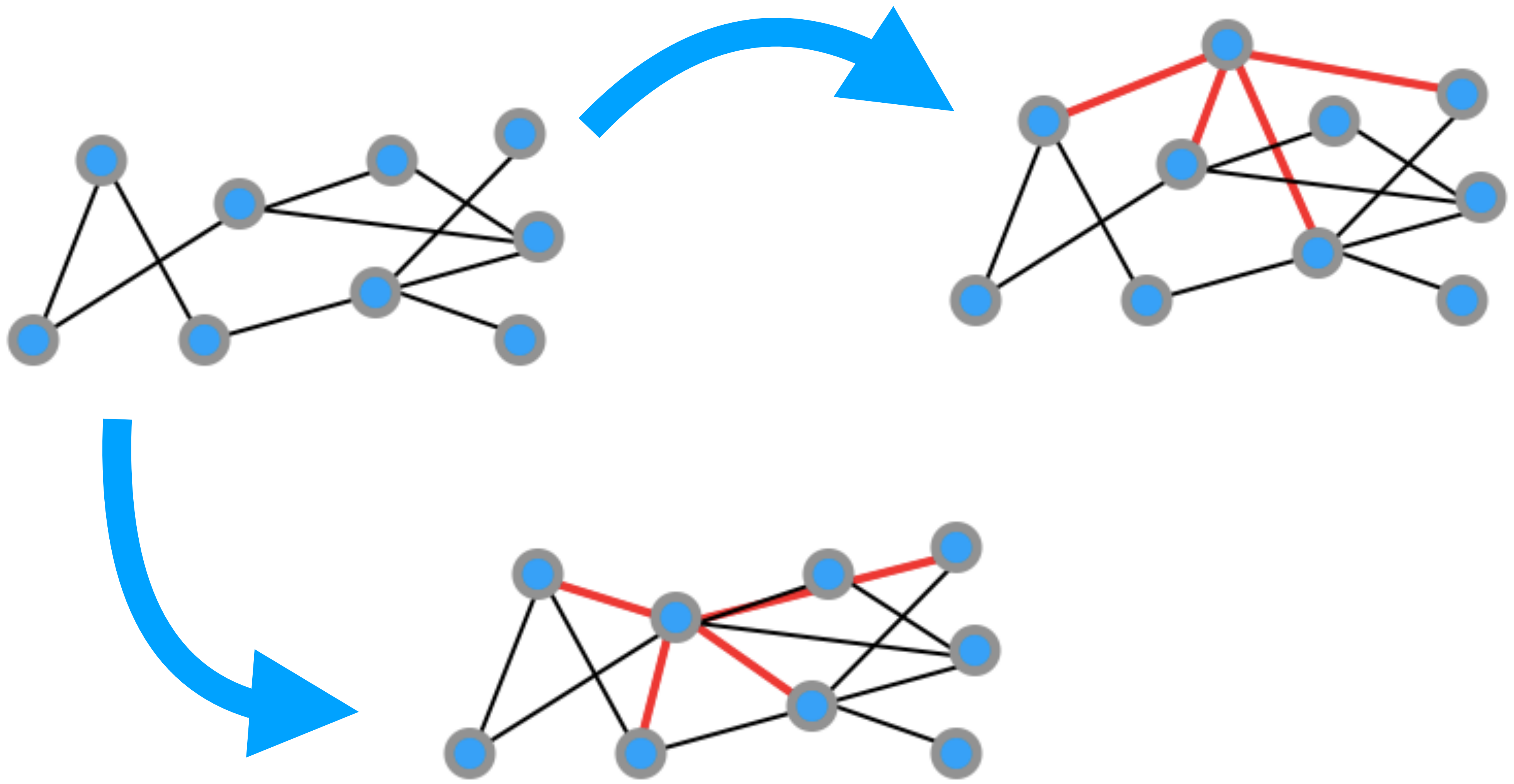

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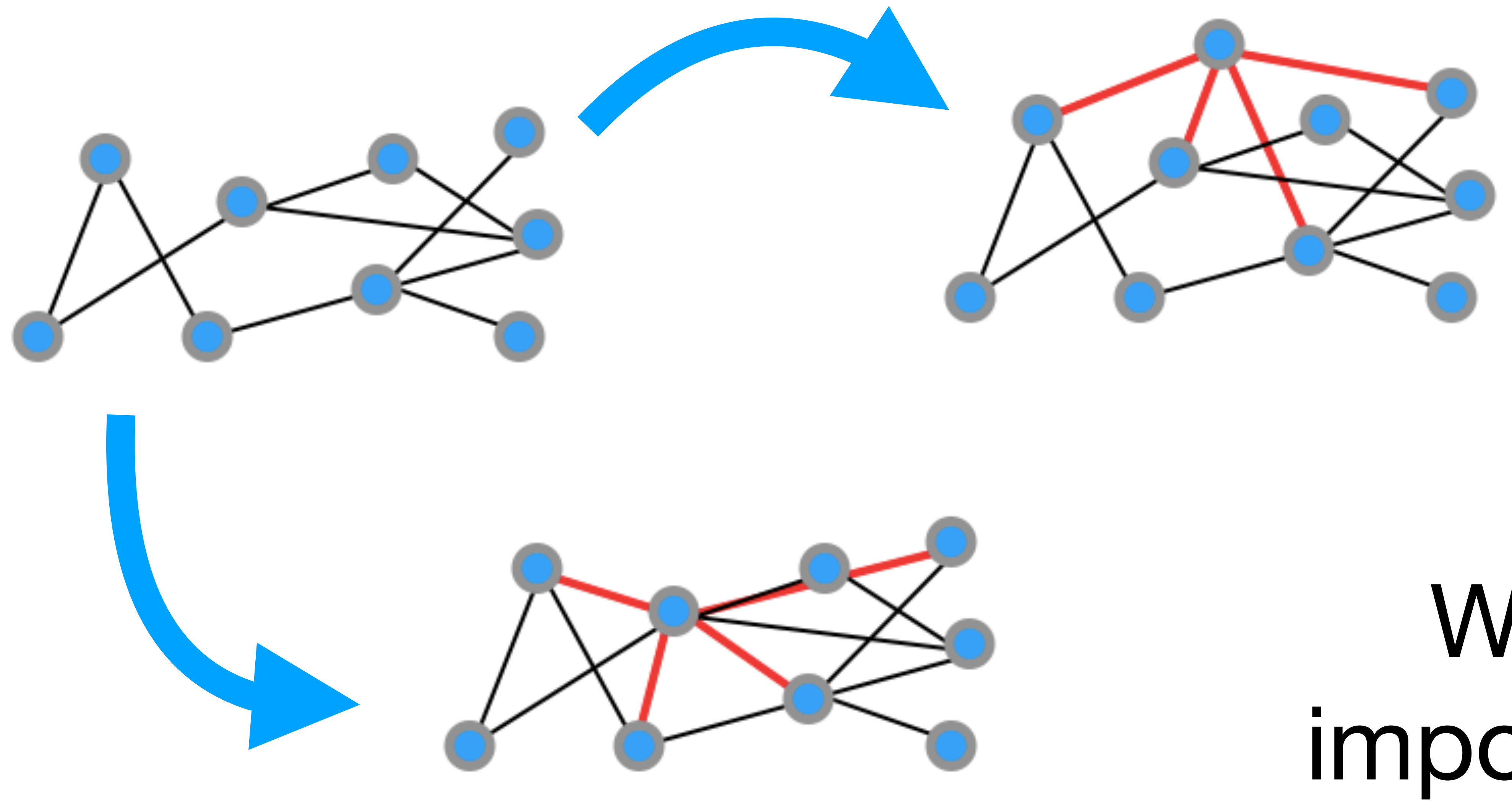

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Weights show importance of each model

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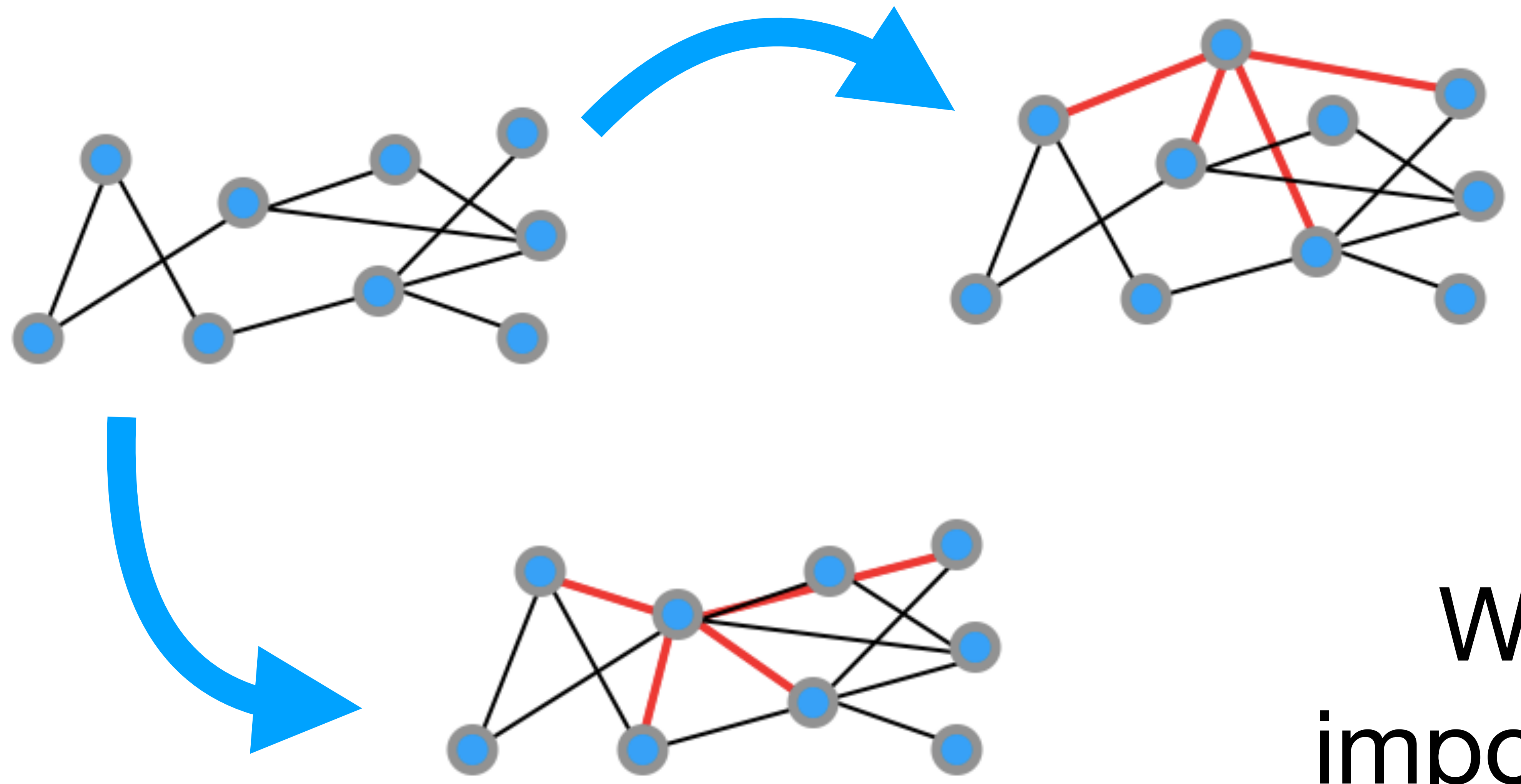

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Weights show importance of each model

This importance may change over time

Artificial data example

$$p_i(t) \propto \begin{cases} k_i^{\alpha} & t \leq T \\ k_i^{\beta} & t > T \end{cases}$$

Preferential attachment with a strength (exponent) that abruptly changes at time T

Model without changepoint found in Krapivsky et al: Connectivity of Growing Random Networks (2000)

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Preferential attachment with a strength (exponent) that abruptly changes at time T

Given we know α and β, can we infer T?

Model without changepoint found in Krapivsky et al: Connectivity of Growing Random Networks (2000)

Example: Nonlinear Preferential Attachment

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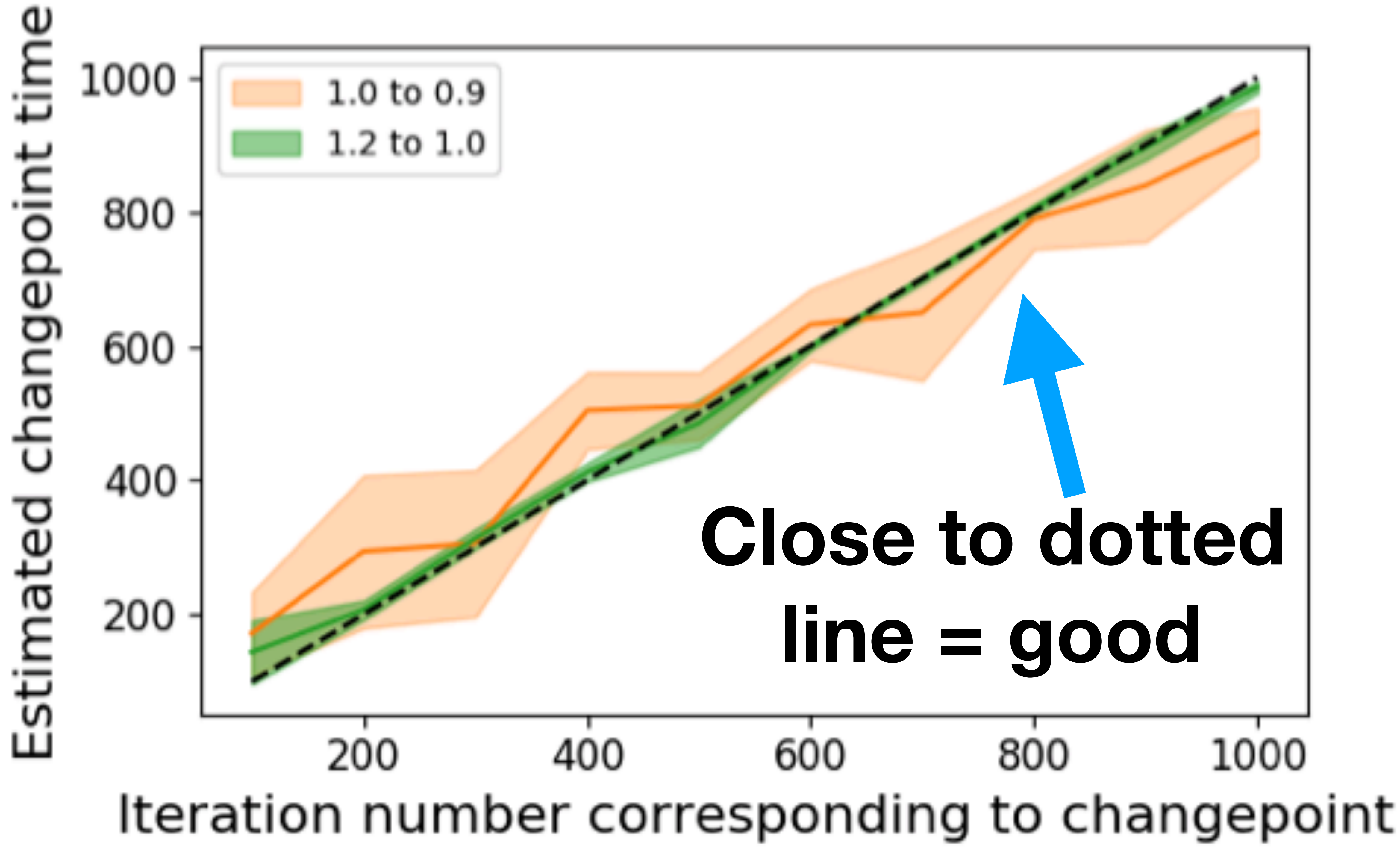

1,000 node network

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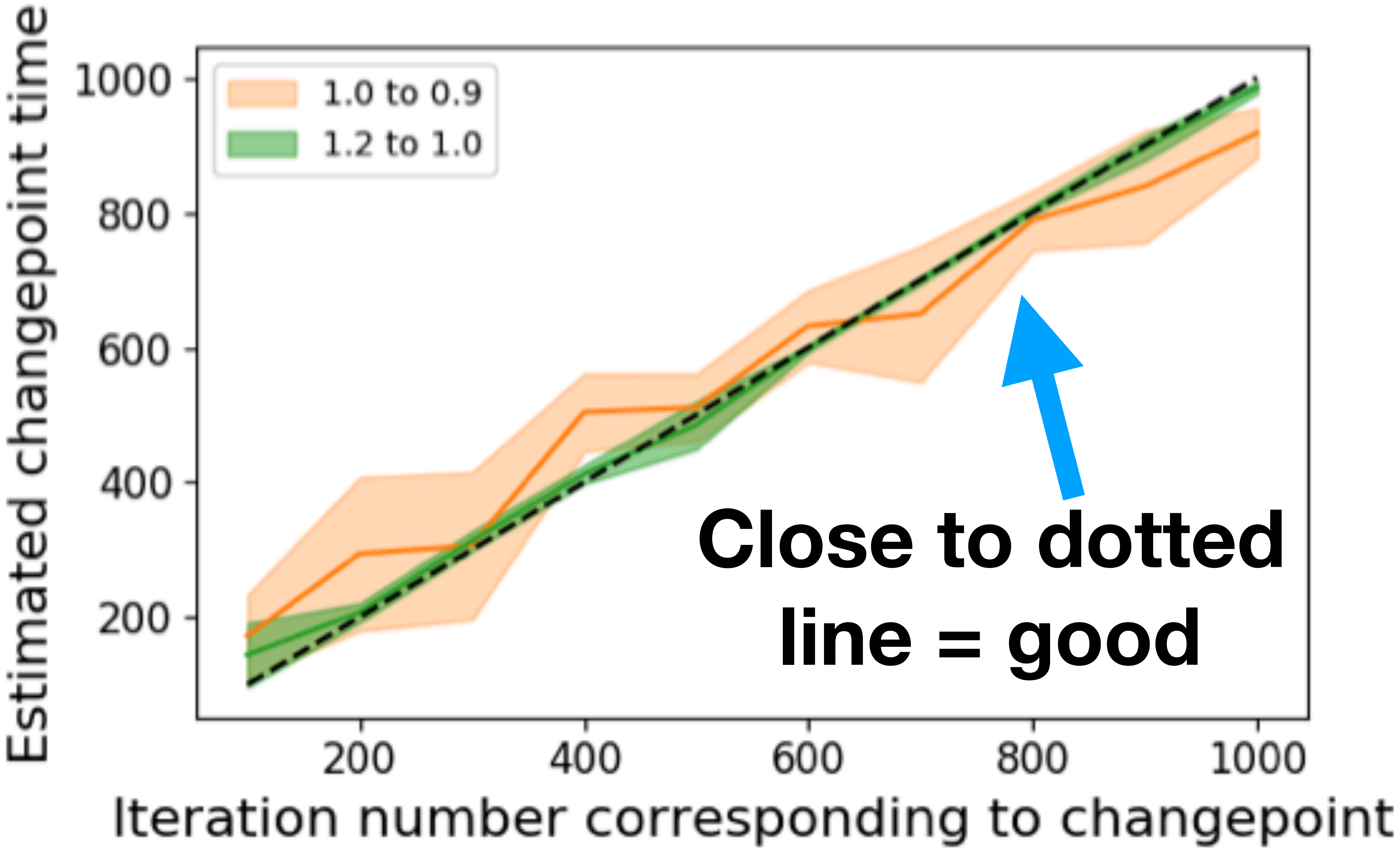

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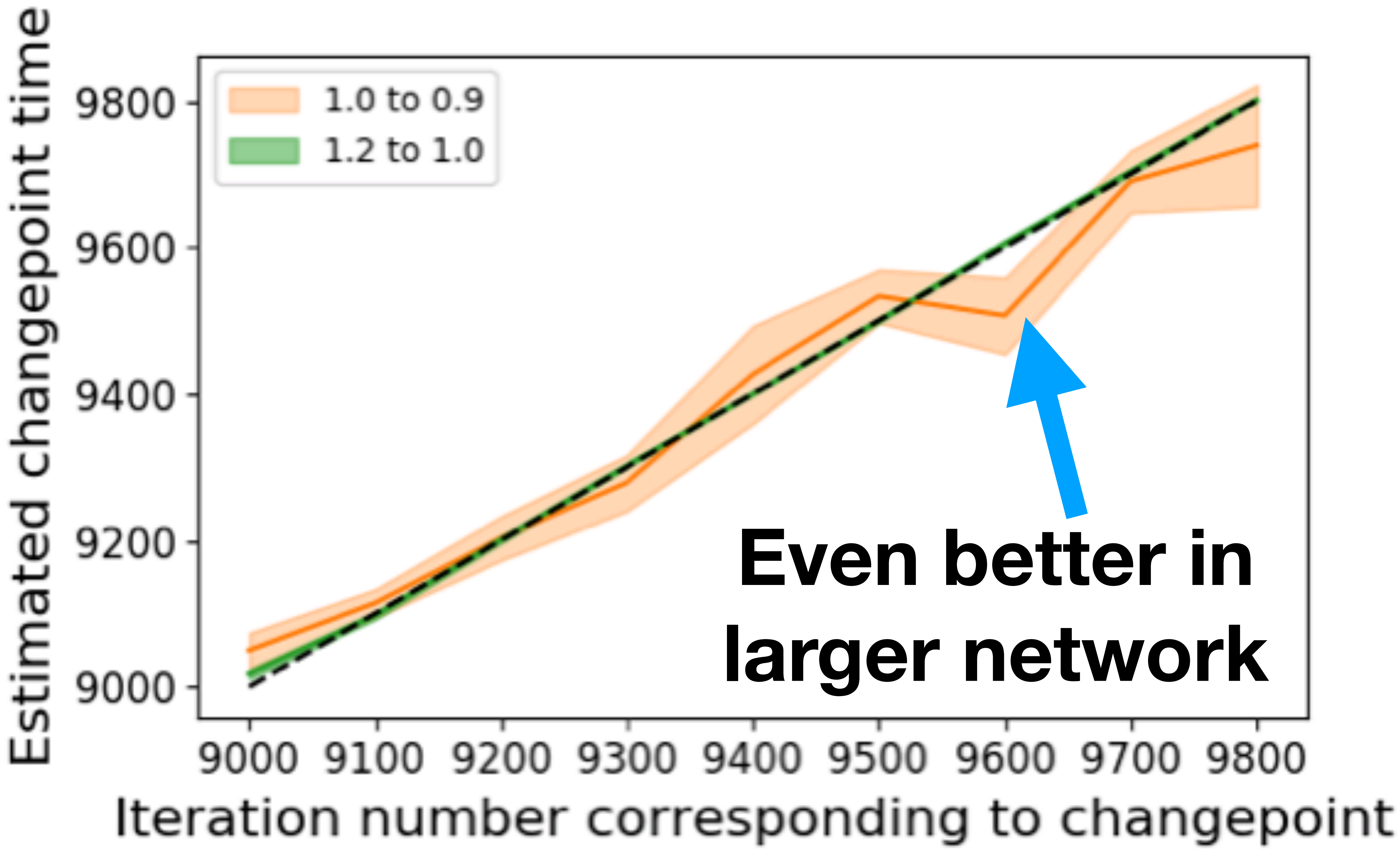
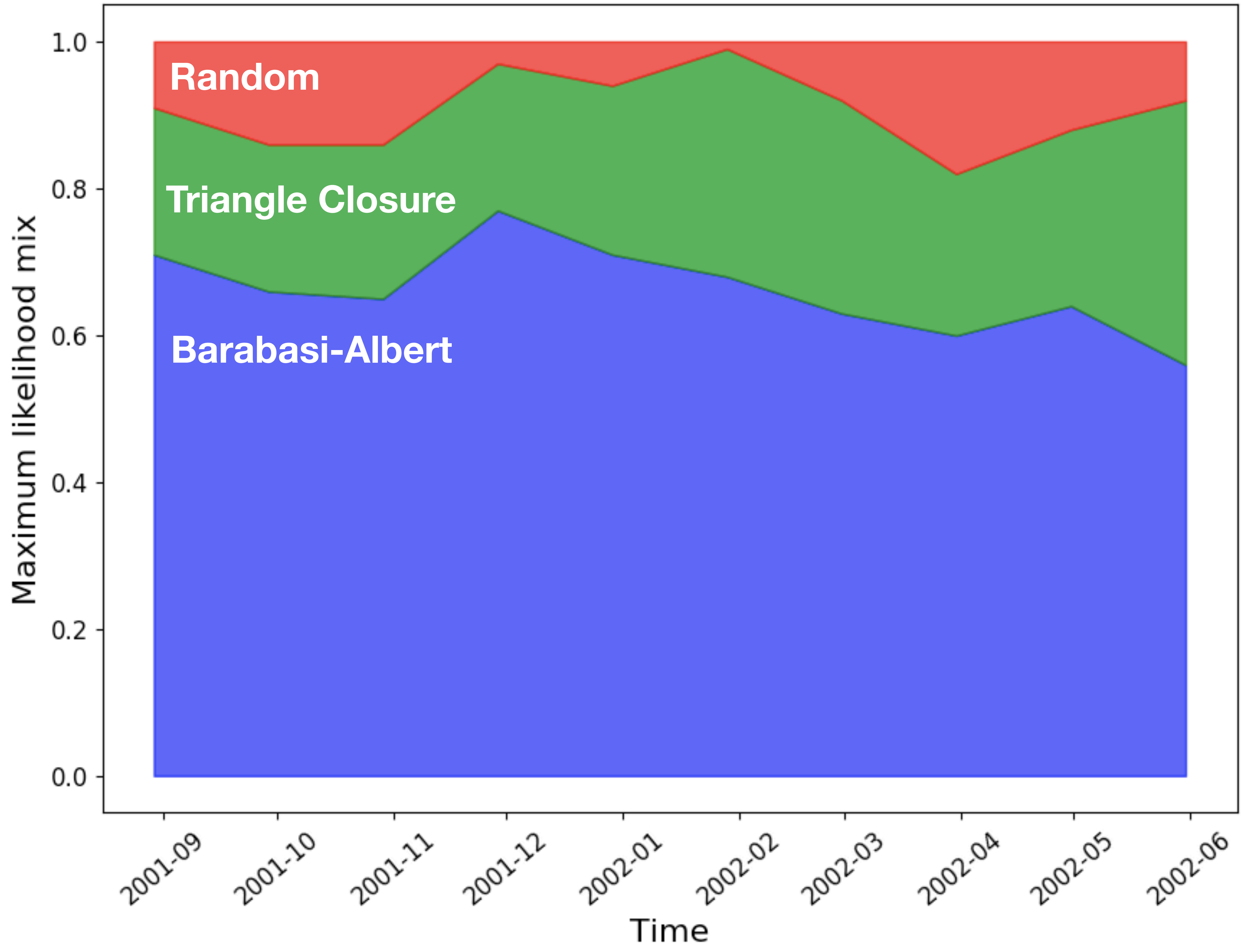


10,000 node network

Enron Revisited

Enron Revisited



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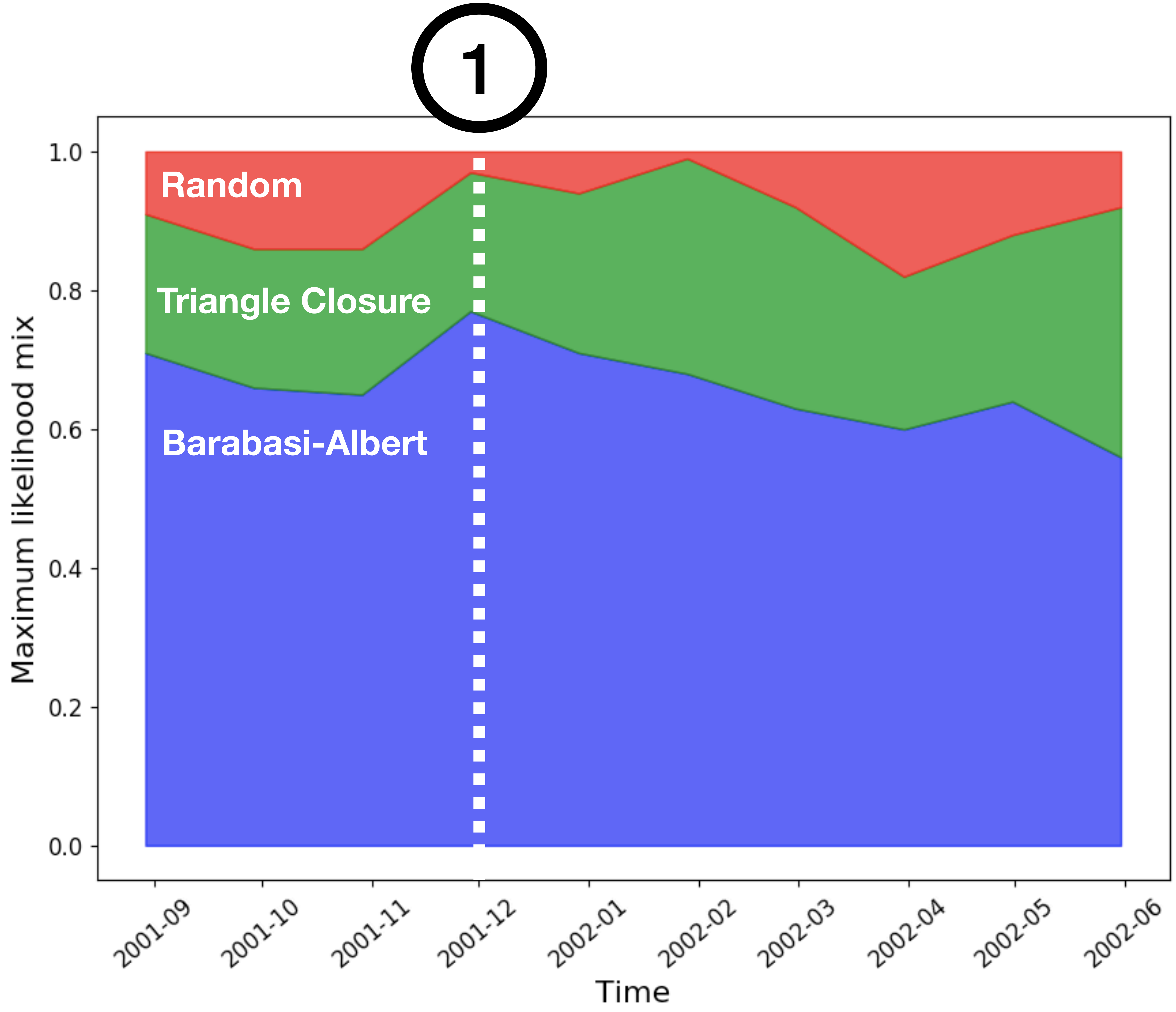
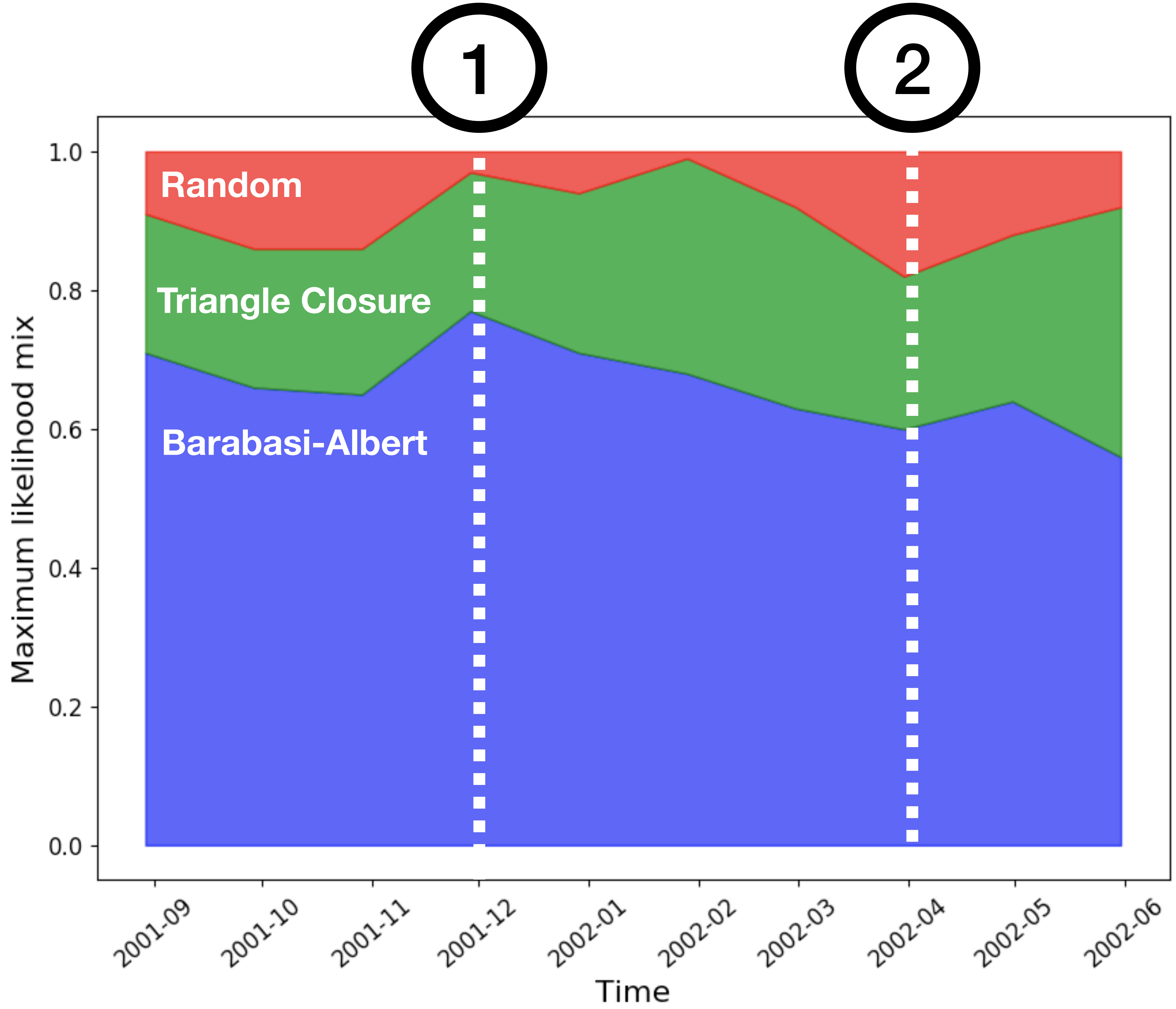


1 **Dec 2, 2001: Enron goes bankrupt, thousands of workers laid off**

Enron Revisited



1 Dec 2, 2001: Enron goes bankrupt, thousands of workers laid off

2 April 9, 2001: Top Enron auditor pleads guilty to obstruction for ordering staff to destroy documents

Takeaways

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- Often a **mixture of mechanisms** better describes a network's growth rather than a single one.
- This mixture may **change over time**, which may tell us about a network's response to events as well as longer term trends.
- Framework for combining these mechanisms gives us a new way of analysing growing networks

Framework for Evolving Topology Analysis

https://github.com/narnolddd/FETA3

Thanks for listening!
Questions?

github.com/narnolddd **n.a.arnold@qmul.ac.uk** **@narnolddd**