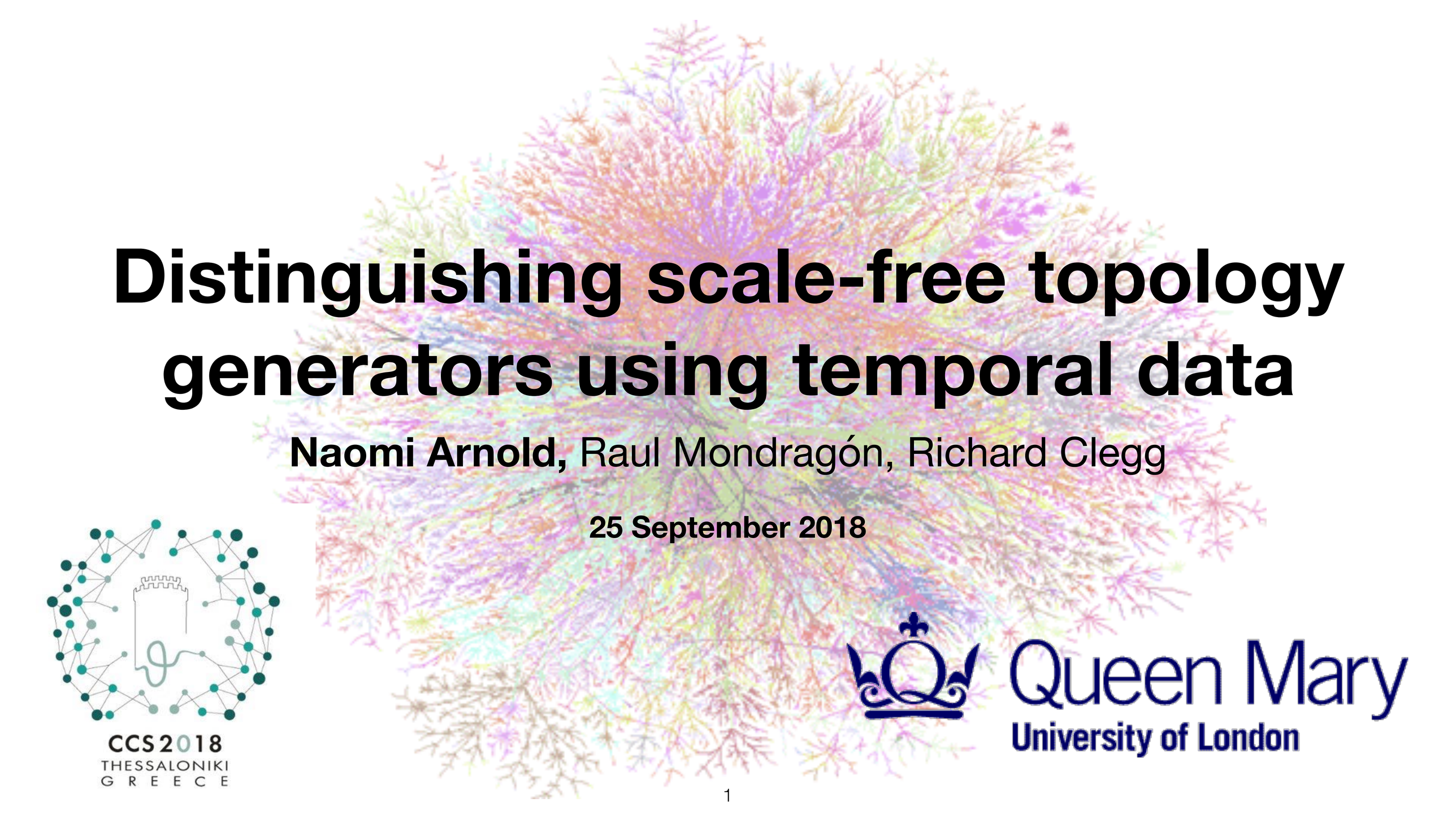


Distinguishing scale-free topology generators using temporal data

Naomi Arnold, Raul Mondragón, Richard Clegg

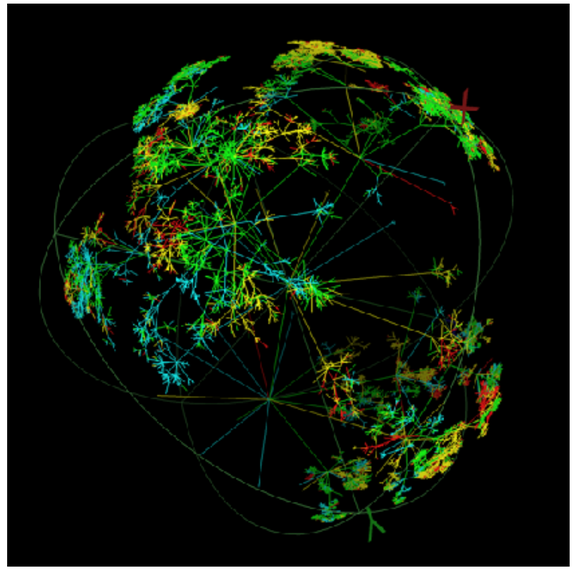
25 September 2018

CCS2018
THESSALONIKI
GREECE

Queen Mary
University of London

Motivation

- The structure of many networks of interest is often dynamic in nature
- Rich temporal data for network topologies is becoming more prevalent
- This allows us to better investigate models for network growth over time



Model for evolving networks

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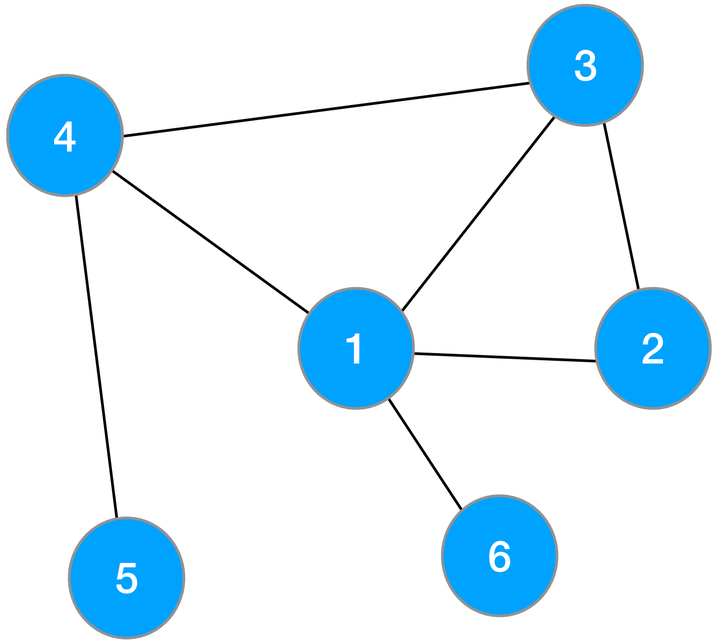
Seed network



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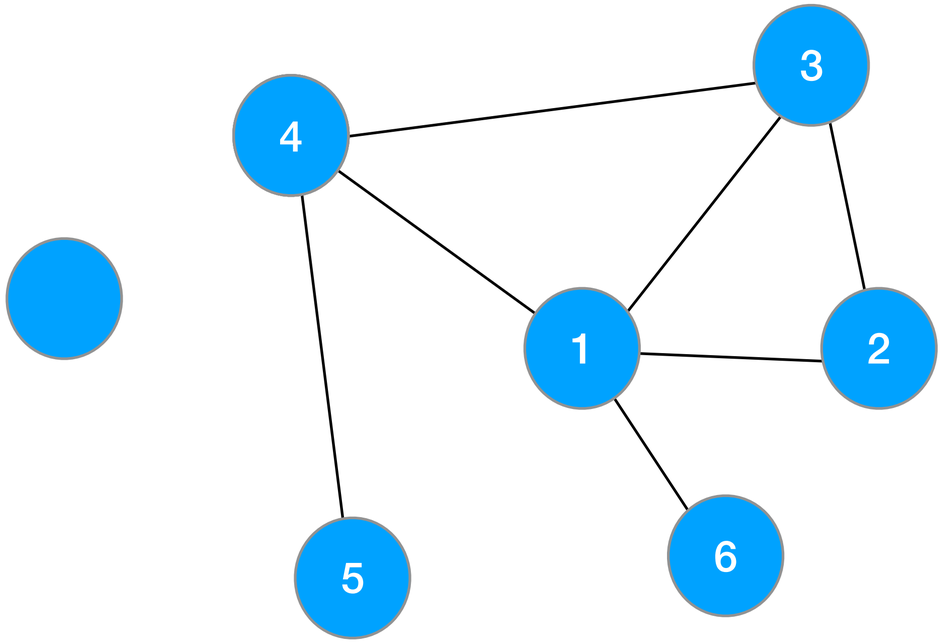

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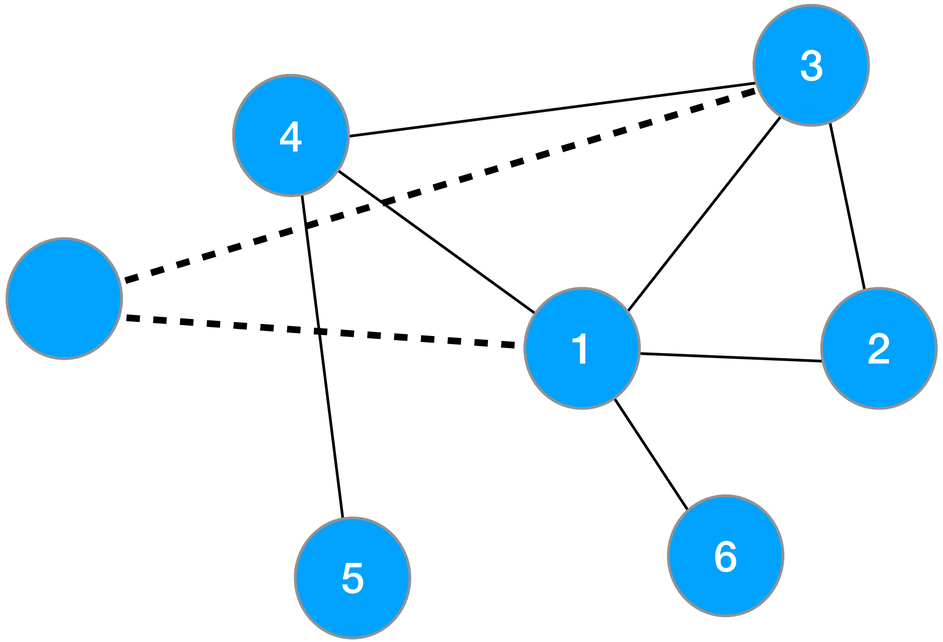

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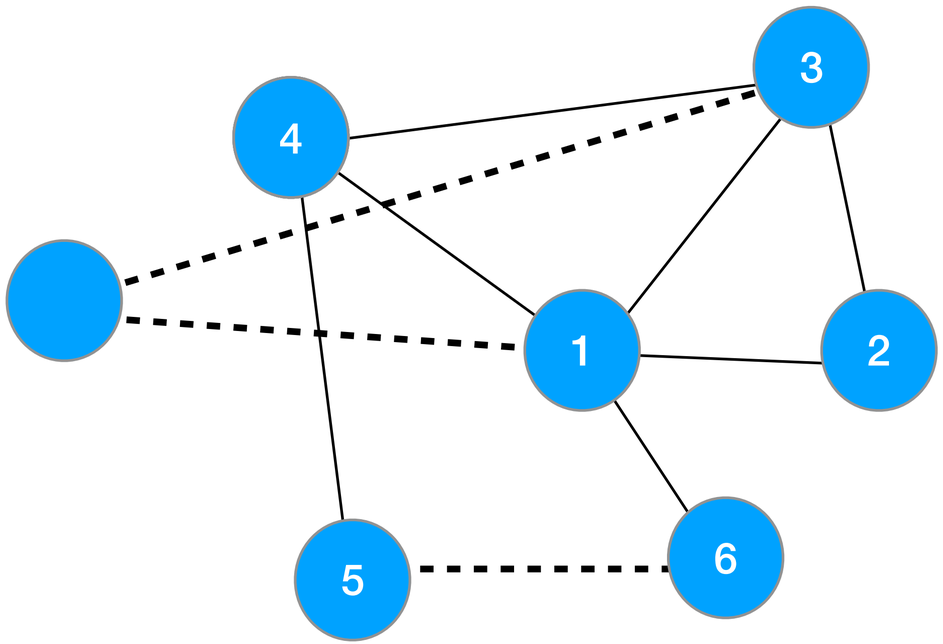

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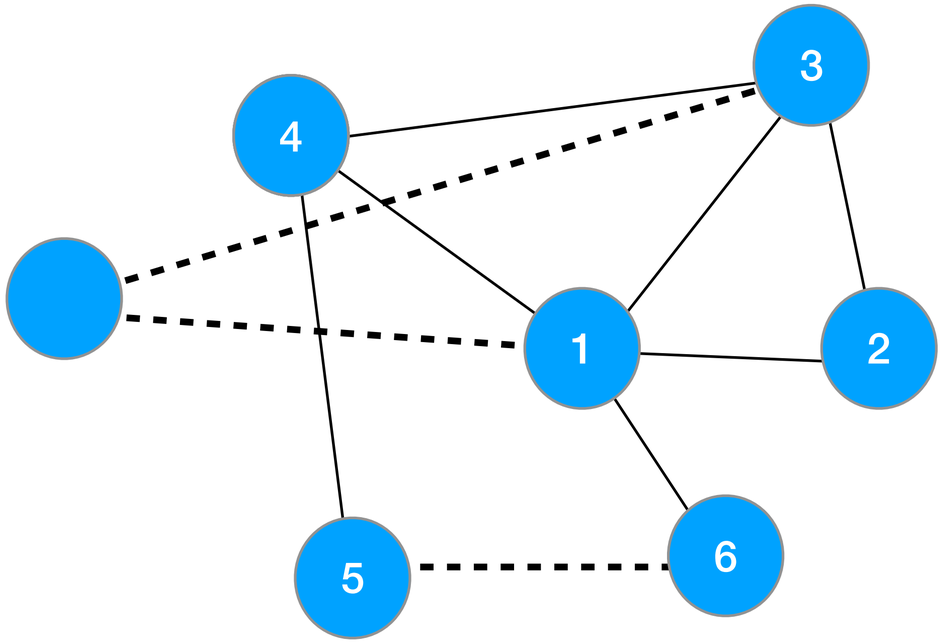

Attachment probabilities

$$p_i$$

corresponding to

$\mathbb{P}$(**choose node** i)

Attachment probabilities

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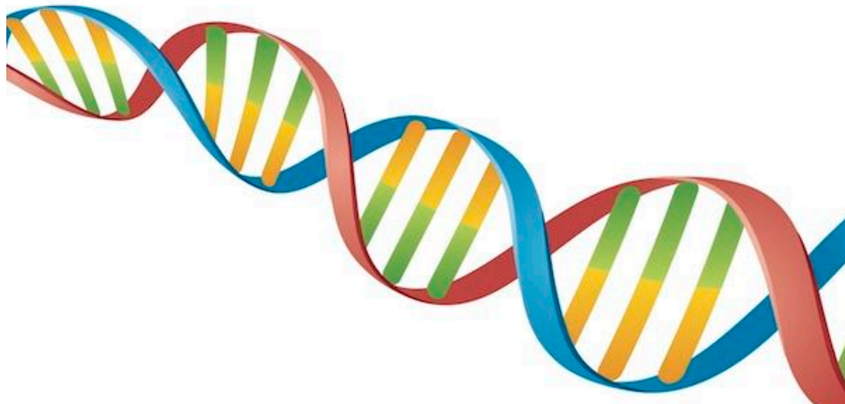
$$p_i \propto f(\eta_i)$$

Function of some other intrinsic node property



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Two different models for scale-free networks

[A.-L. Barabasi, R. Albert 1999: Emergence of scaling in random networks]

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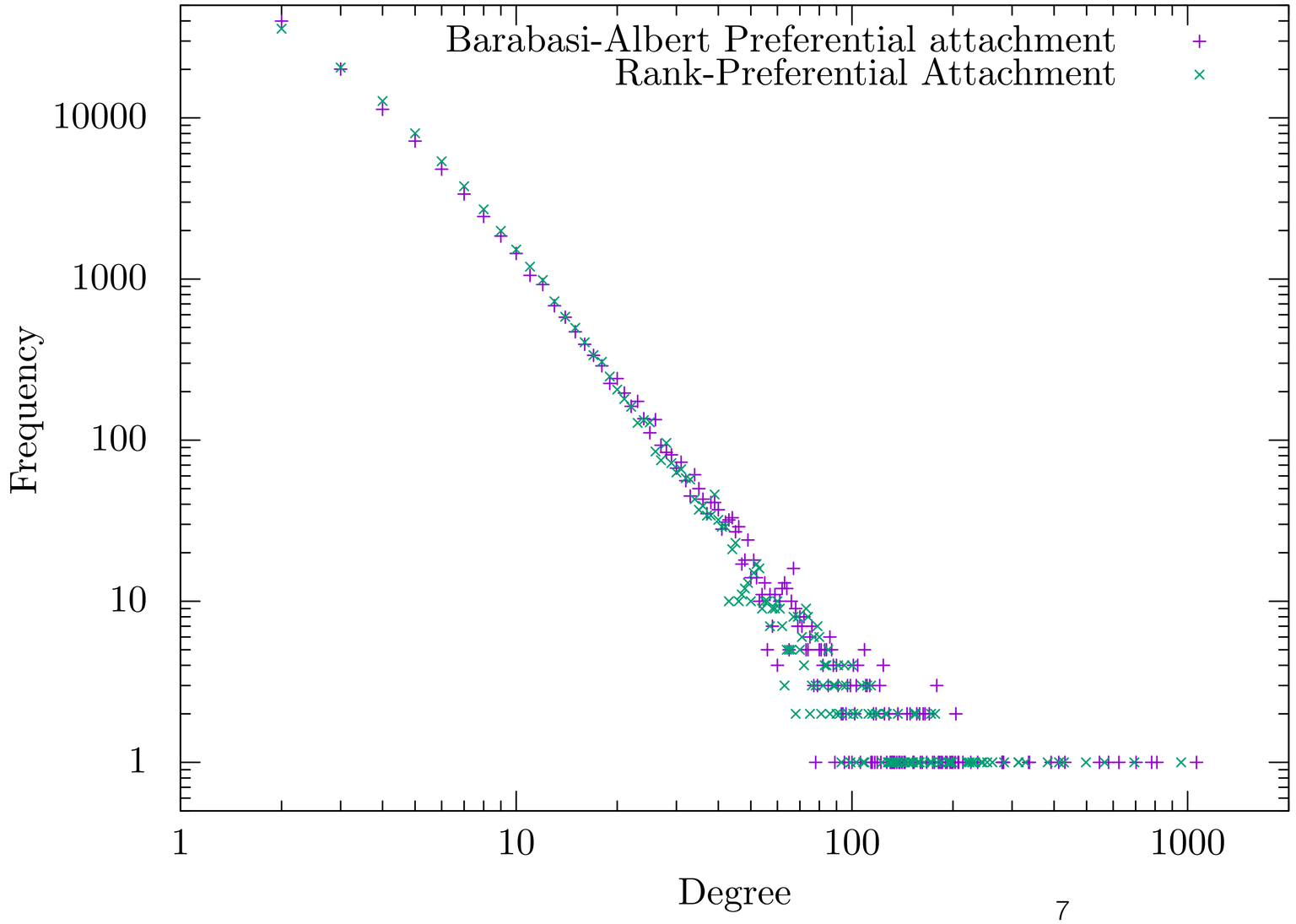
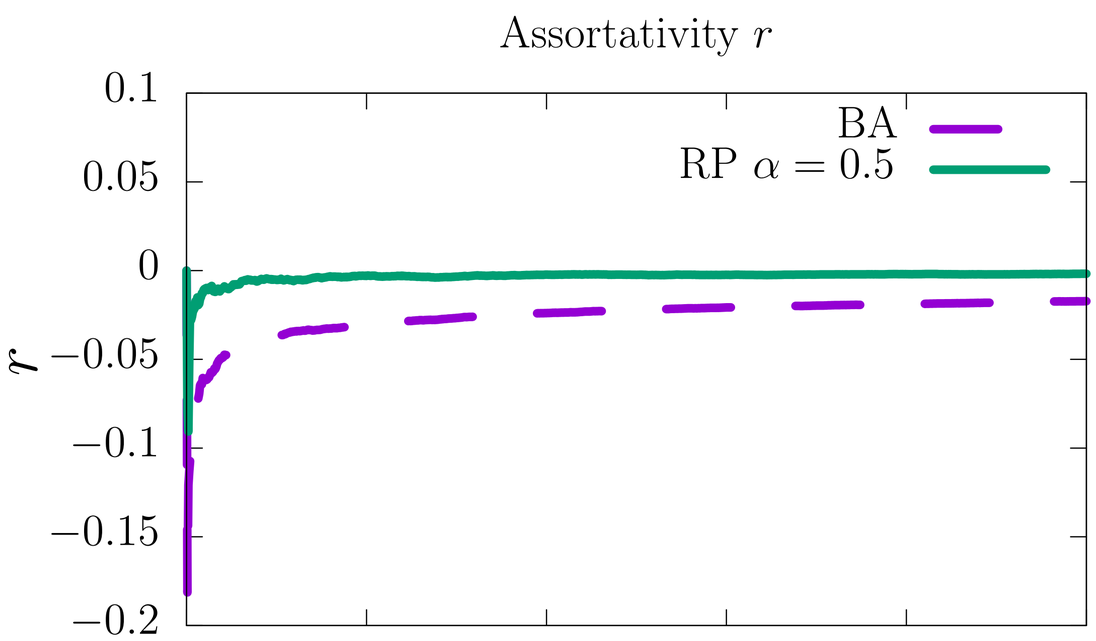
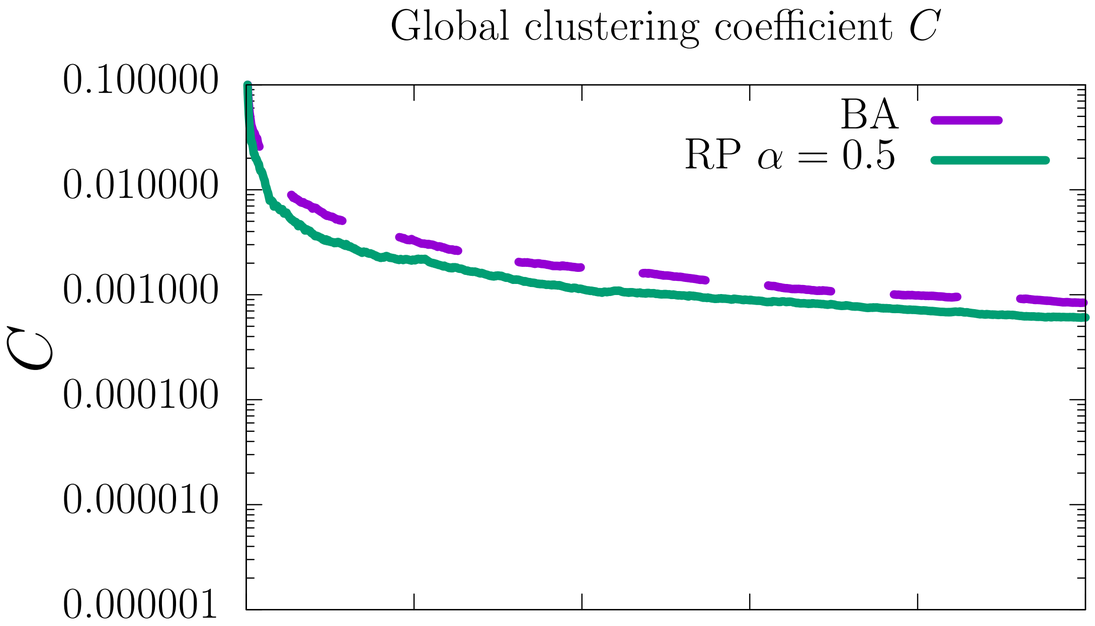
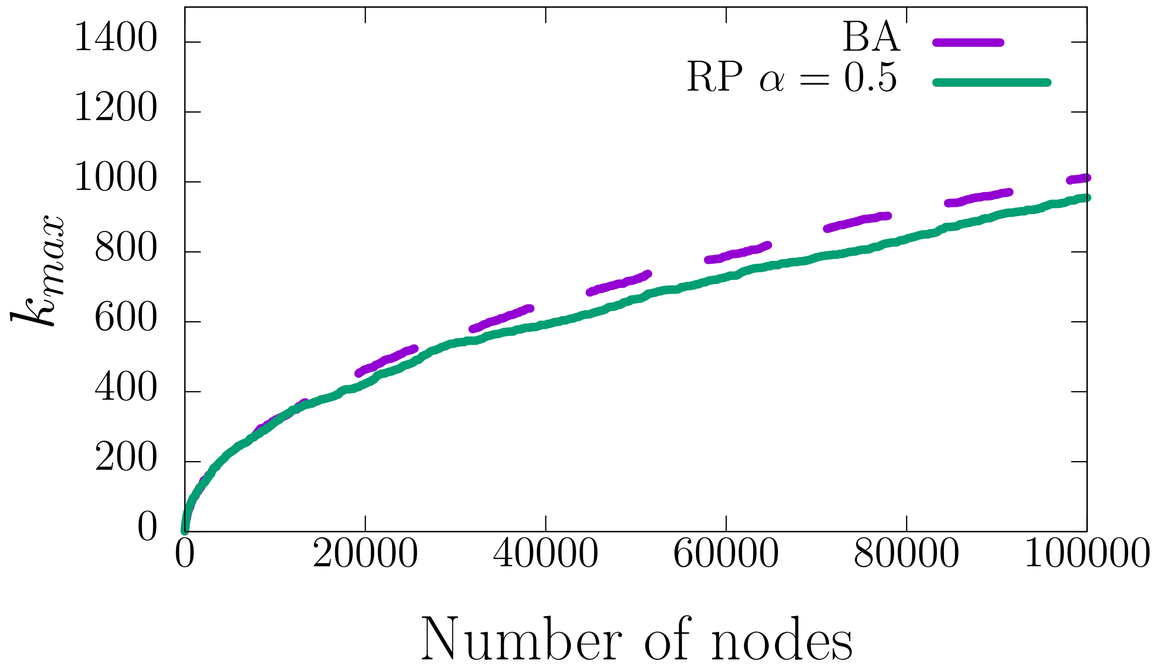
Degree distribution

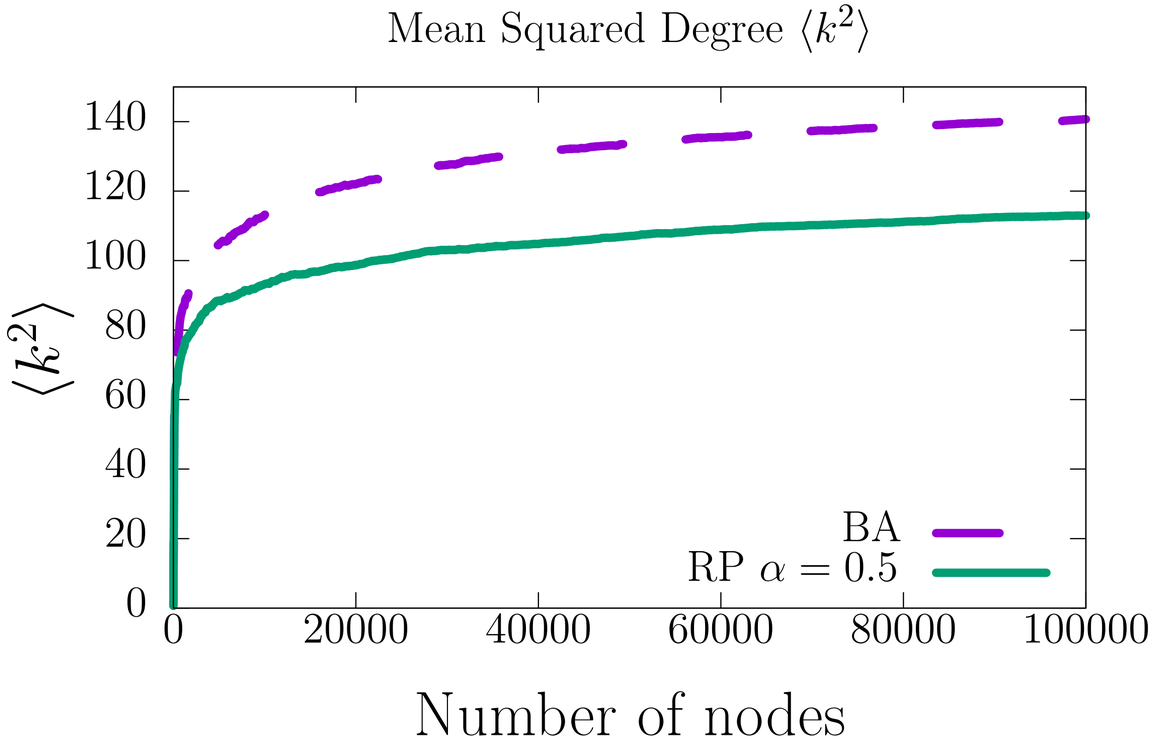


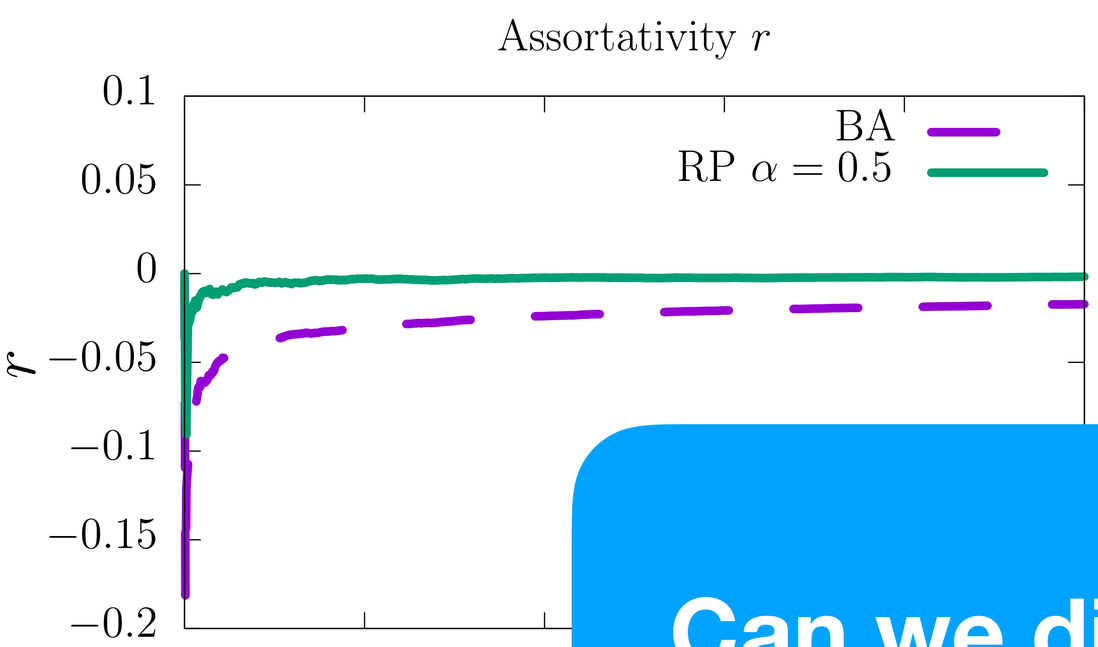
Network of 100,000 nodes generated from the Barabási-Albert and Rank Preference models, each new node having initially 3 neighbours

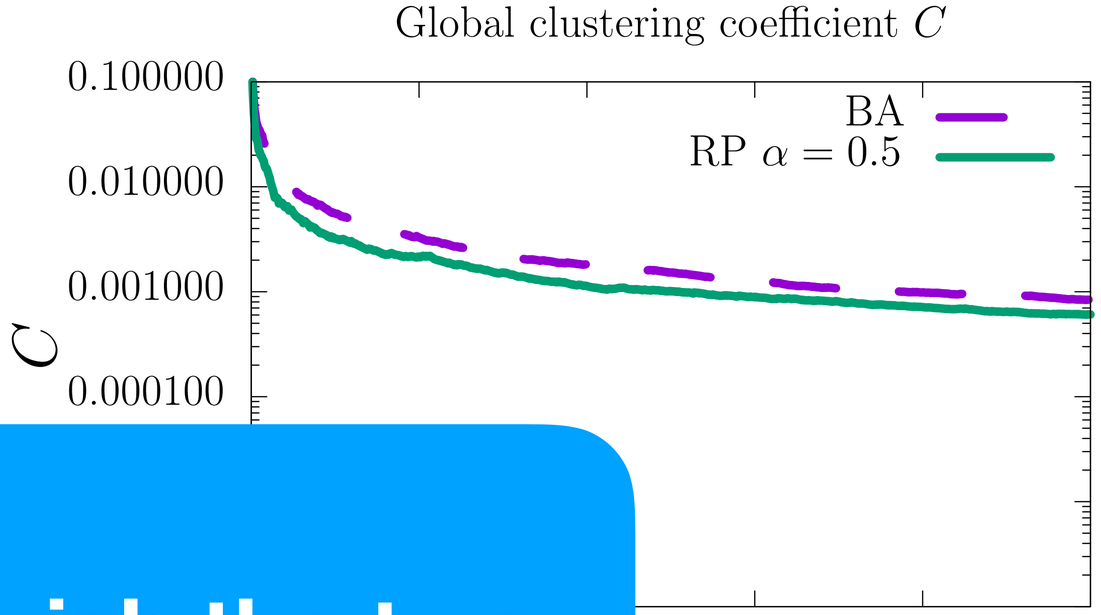

Assortativity r
BA
RP $\alpha = 0.5$
r
0.1
0.05
0
−0.05
−0.1
−0.15
−0.2


Global clustering coefficient C
BA
RP $\alpha = 0.5$
C
0.100000
0.010000
0.001000
0.000100
0.000010
0.000001

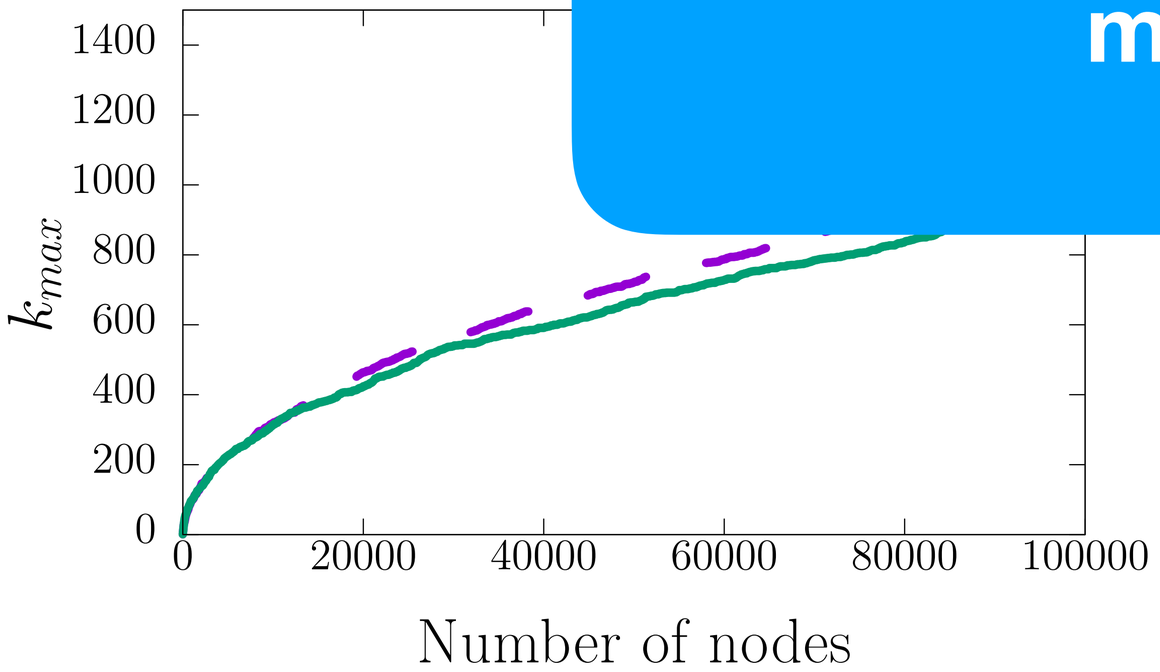

Maximum degree k_{max}
BA
RP $\alpha = 0.5$
k_{max}
1400
1200
1000
800
600
400
200
0
0
20000
40000
60000
80000
100000
Number of nodes

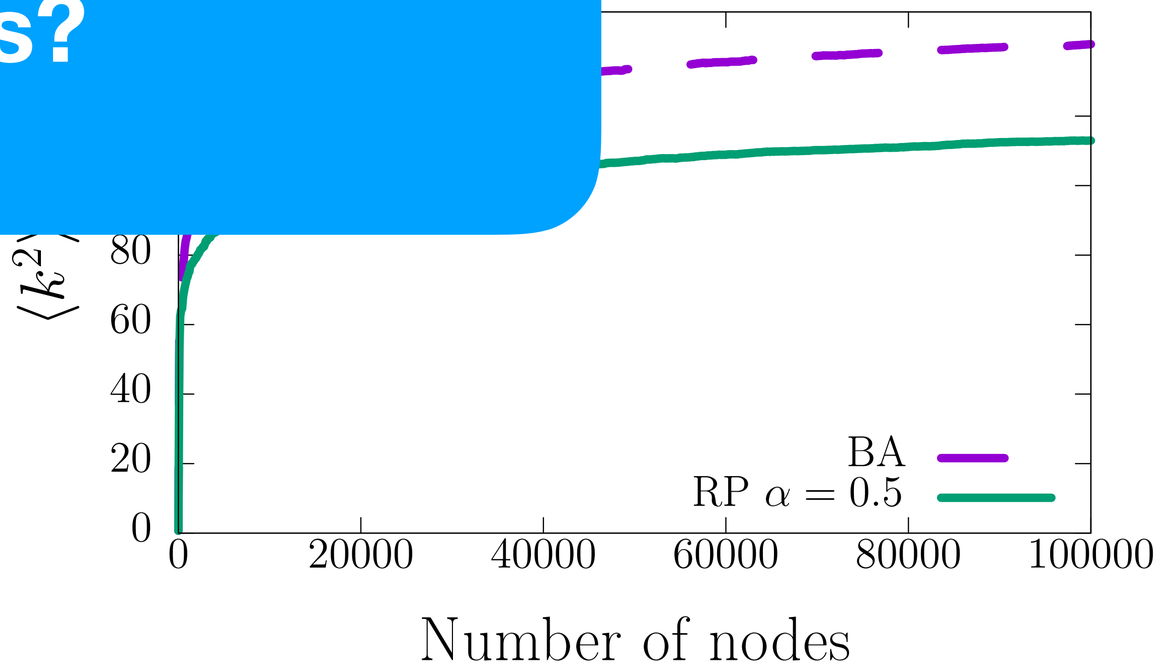

Mean Squared Degree $\langle k^2 \rangle$
BA
RP $\alpha = 0.5$
$\langle k^2 \rangle$
140
120
100
80
60
40
20
0
0
20000
40000
60000
80000
100000
Number of nodes





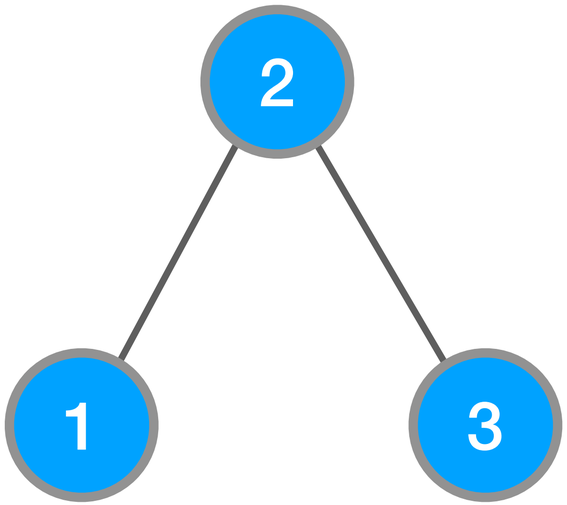
Can we distinguish the two models?





Different approach: Model likelihood

Likelihood of model given observation = probability of seeing observation given model



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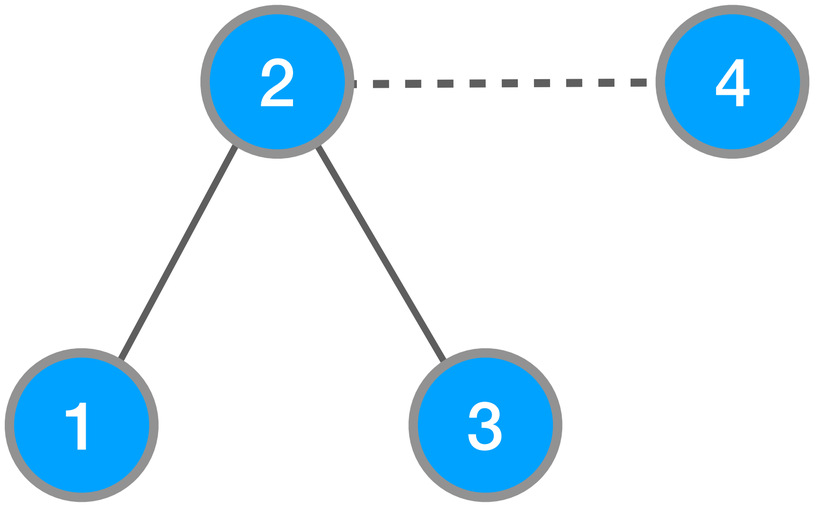

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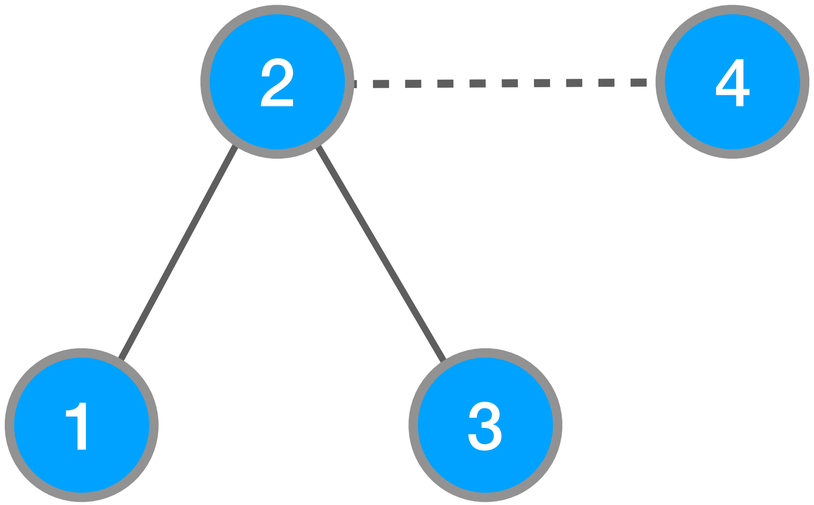

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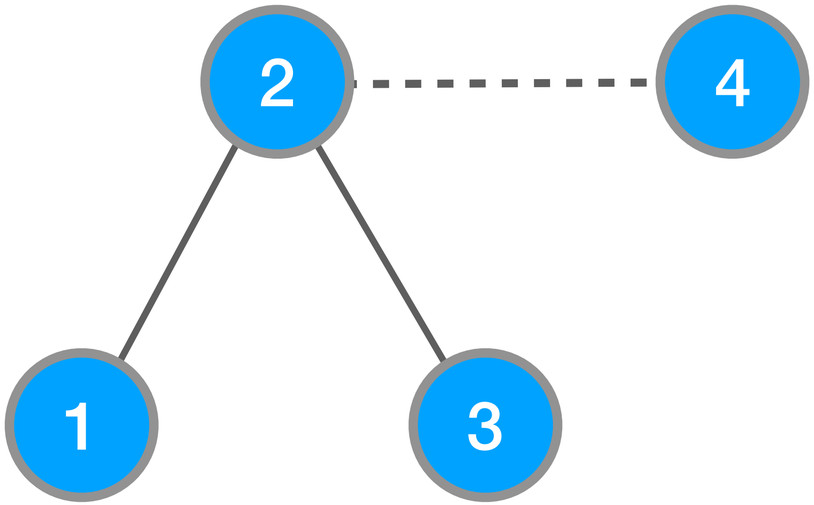

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Likelihood of BA preferential attachment model given by:

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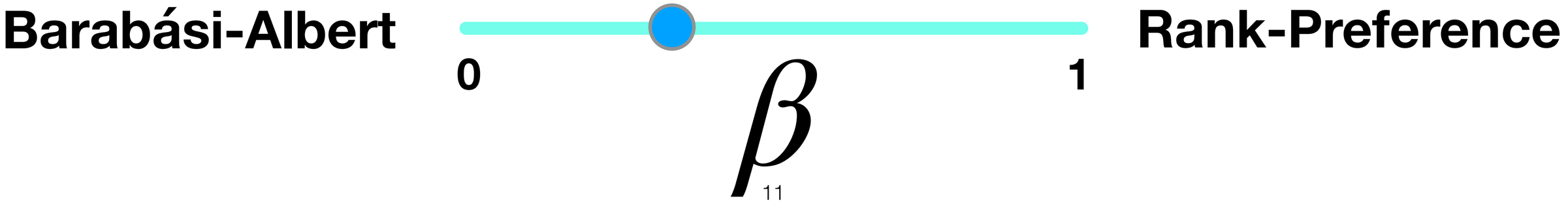
- For models with parameters, can find maximum likelihood estimators for params

Model Mixtures

Hypothesis: network growth likely to be governed by a mixture of mechanisms, not just one

Example: Mixture of BA and rank preference model

$$p_i = (1 - \beta)p_i^{BA} + \beta p_i^{RP}$$



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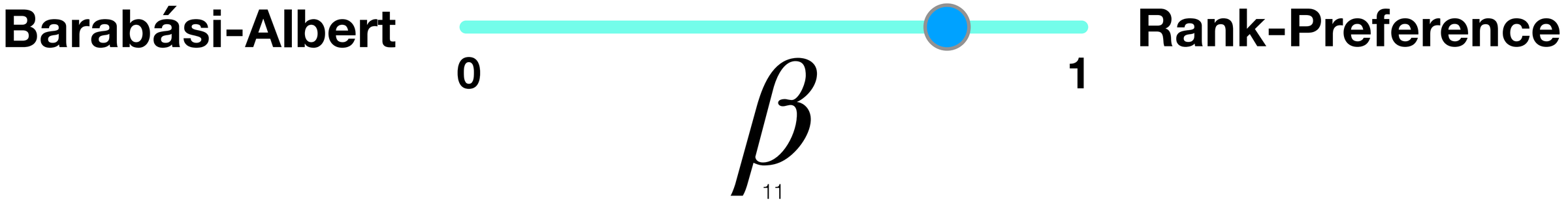

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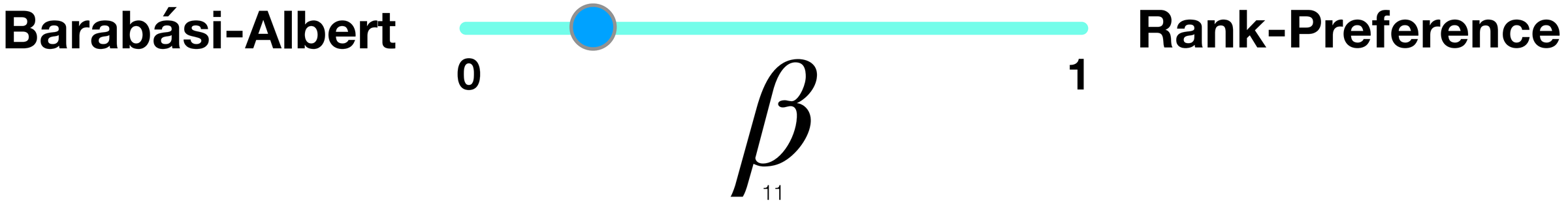

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We can accurately recover the proportion β as an MLE!

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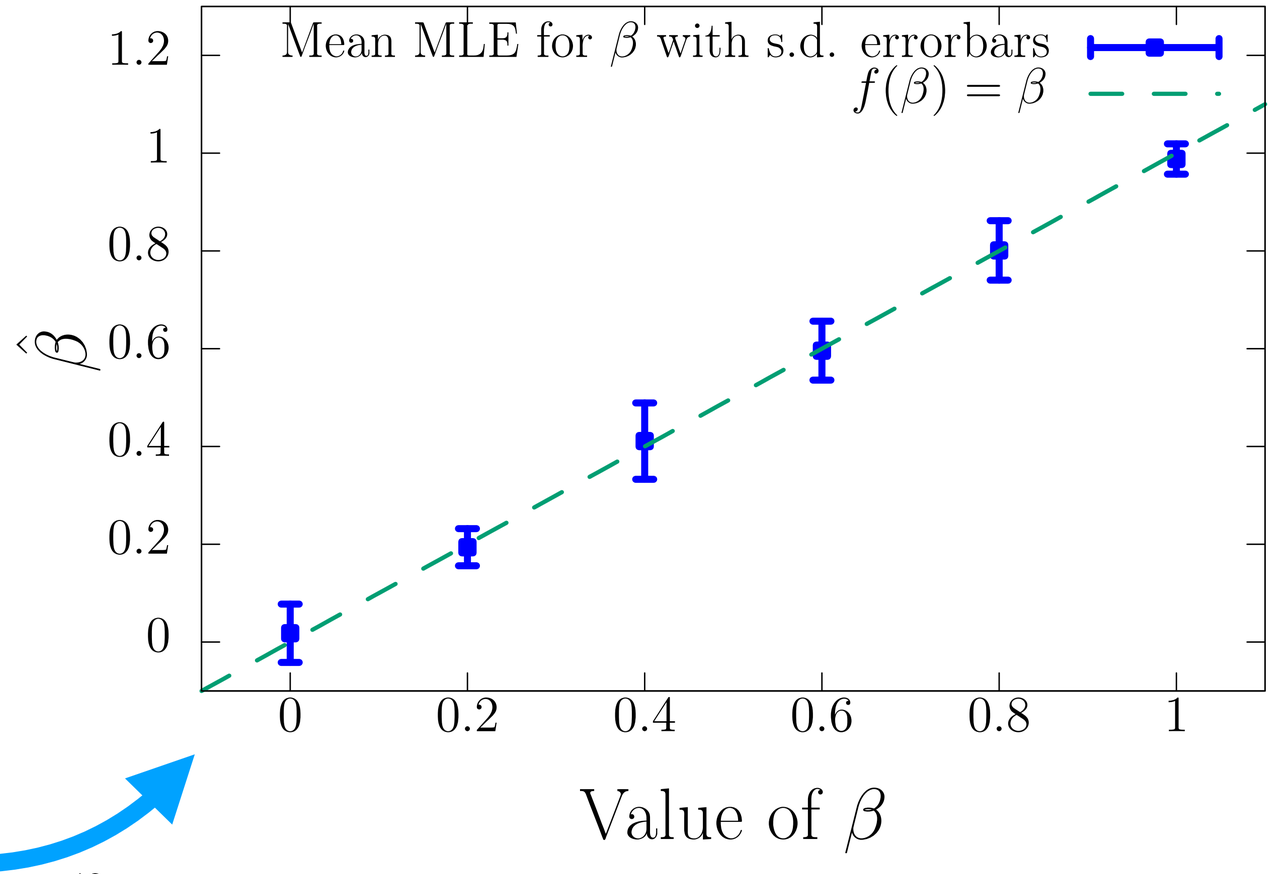

Real data example: Math Overflow Social Network

Dataset from [A. Paranjape, A. Benson, J. Leskovec 2017: Motifs in temporal networks]

Real data example: Math Overflow Social Network

- Q&A site for mathematical problems
- Nodes are users
- An undirected edge between node A and B if A answers a question by B, A comments on B's answer or question
- Multiple edges collapsed
- Models components tested: BA, static rank preference

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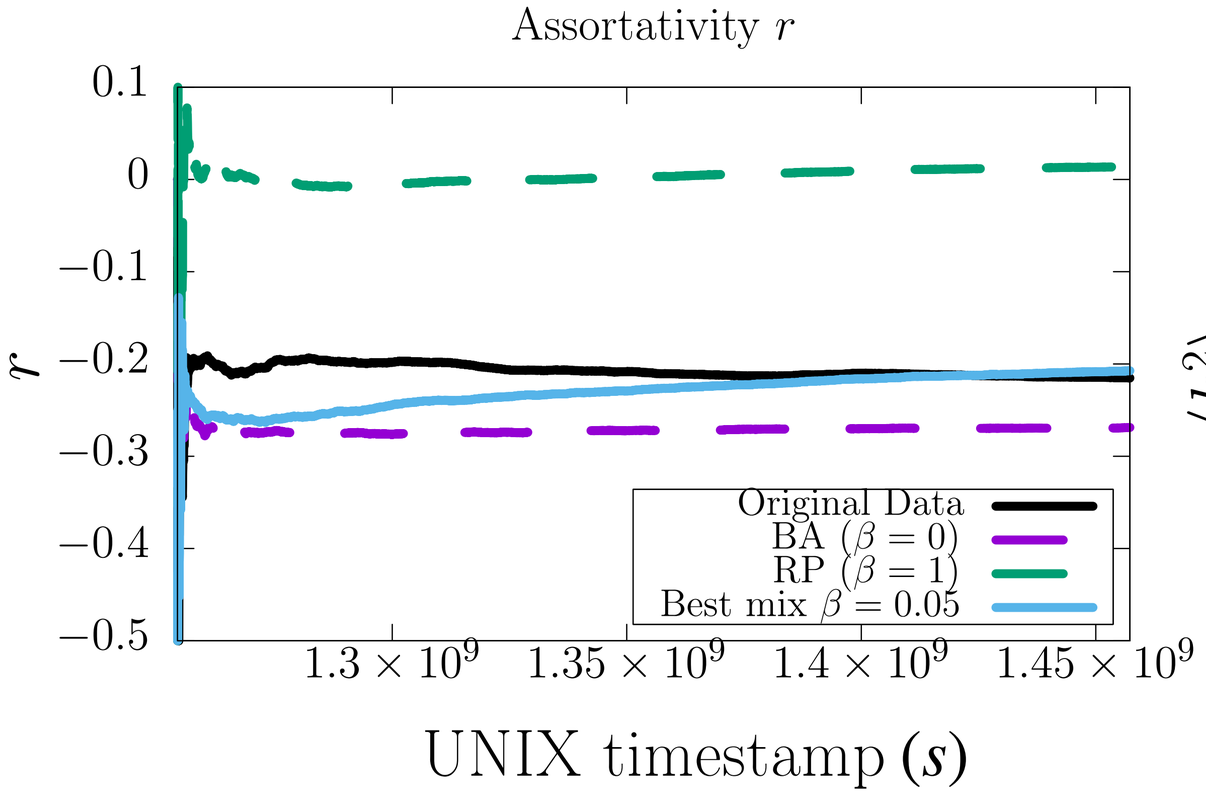

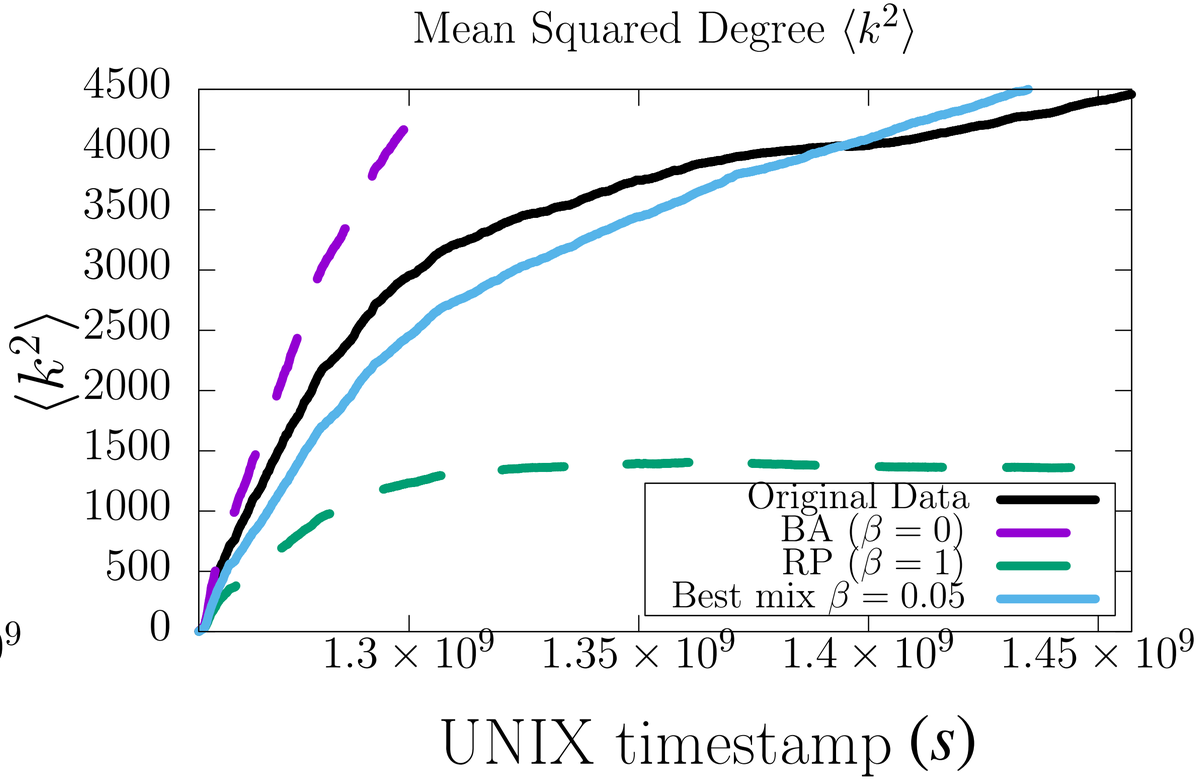

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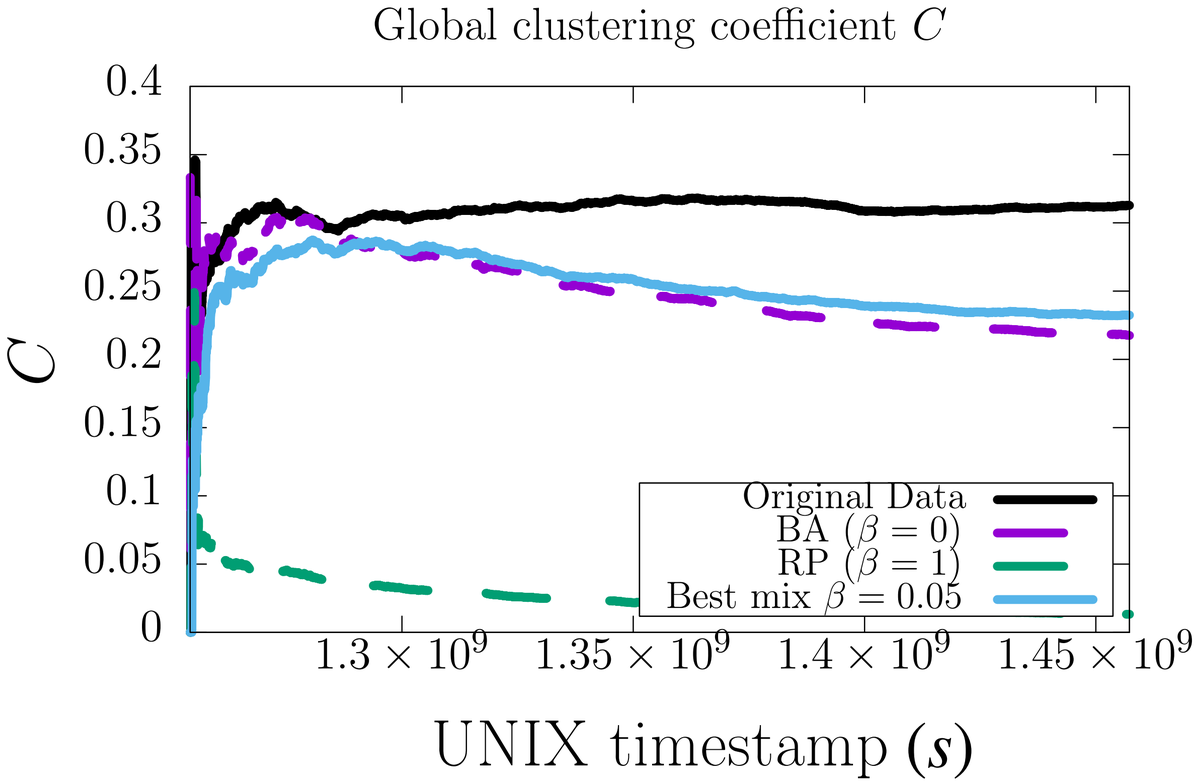

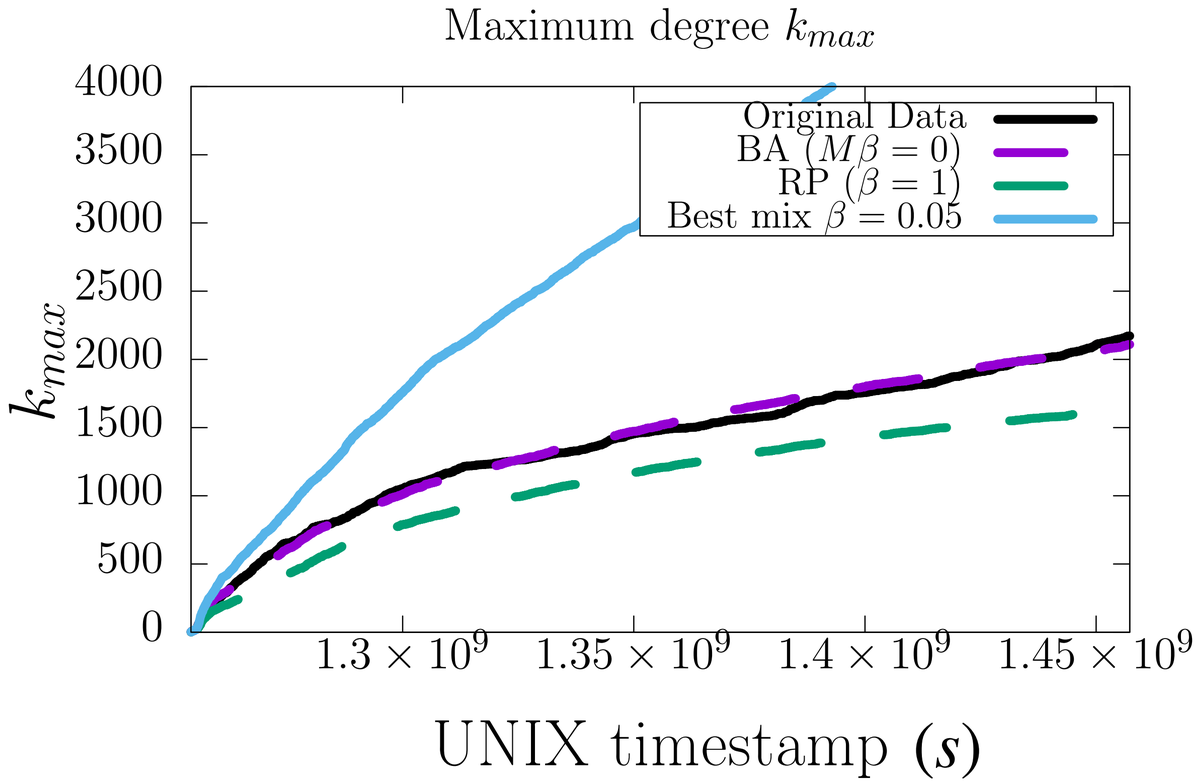

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Real data example: Routeviews AS topology dataset

- Timestamped dataset: nodes autonomous systems and links represent peering relationship
- Model components tested: random/uniform model, positive-feedback preference model

Note - positive-feedback preference model gives:

$$p_i \propto k_i^{1+\delta \log_{10} k_i}$$

PFP model: [S. Zhou, R. Mondragón 2004: Accurately modelling the Internet Topology]

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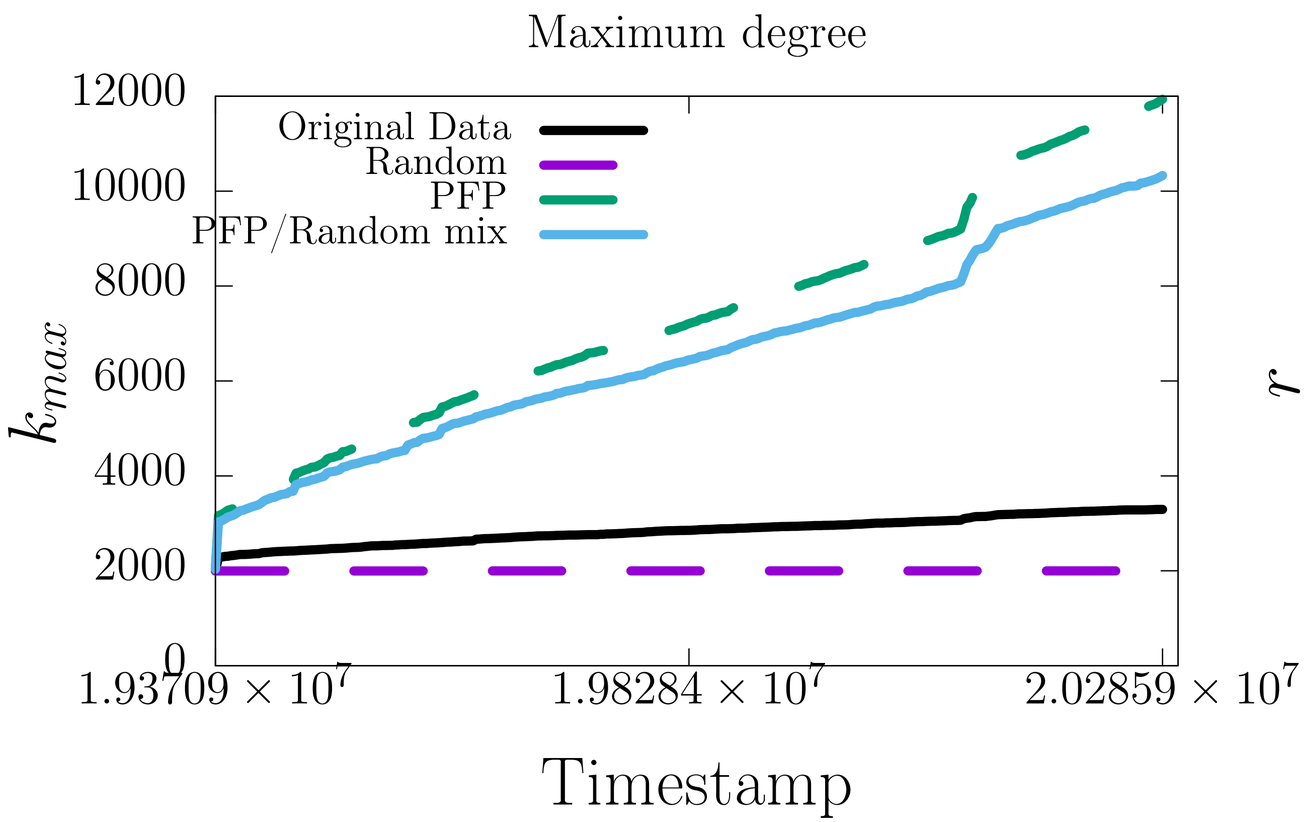

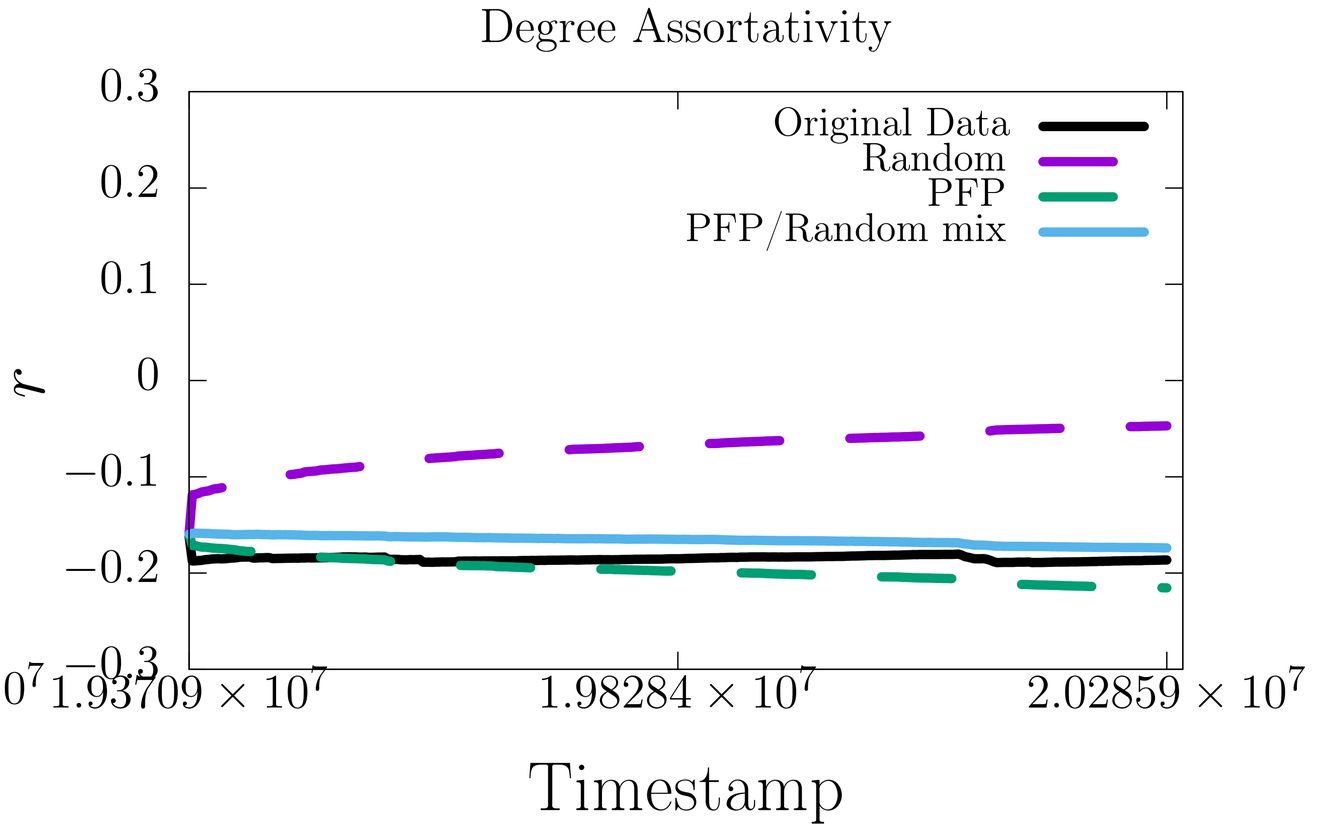

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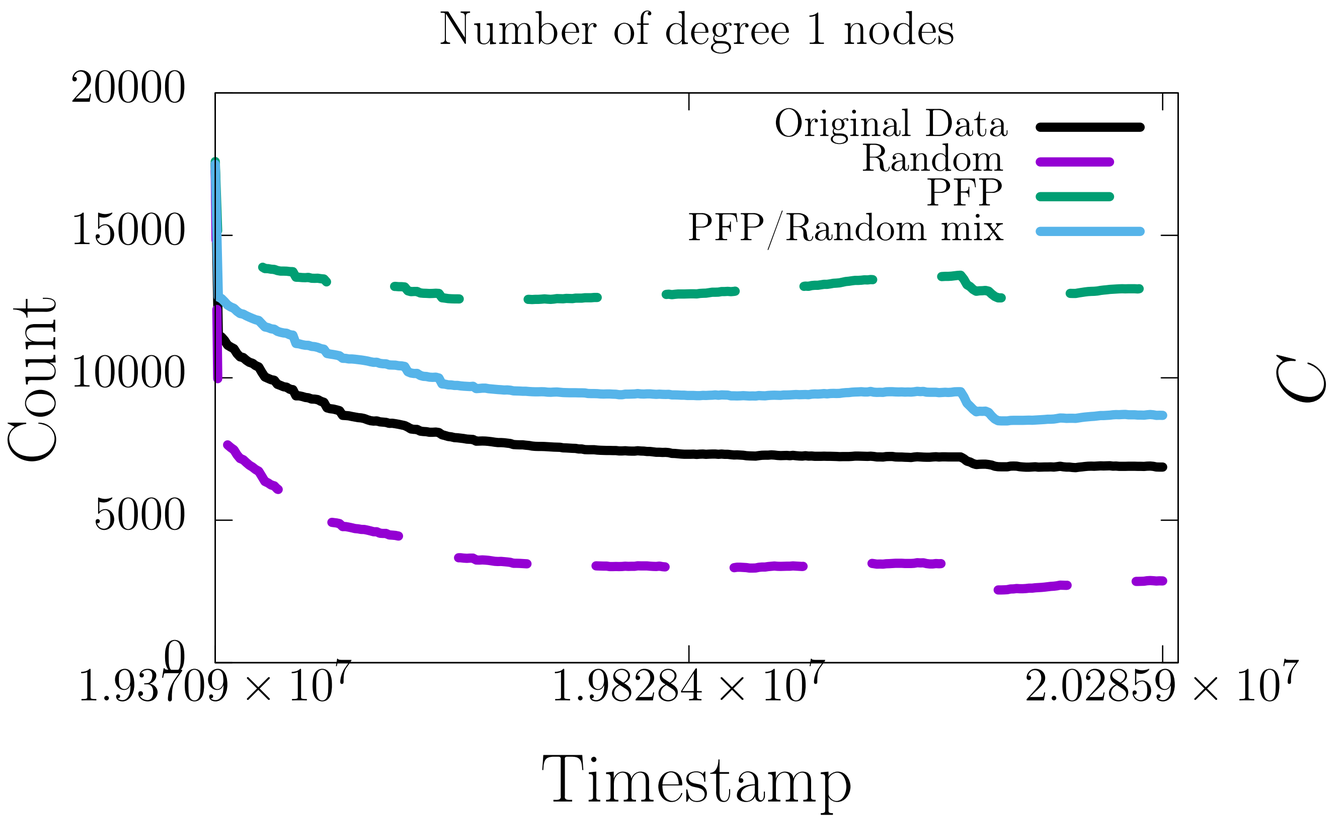

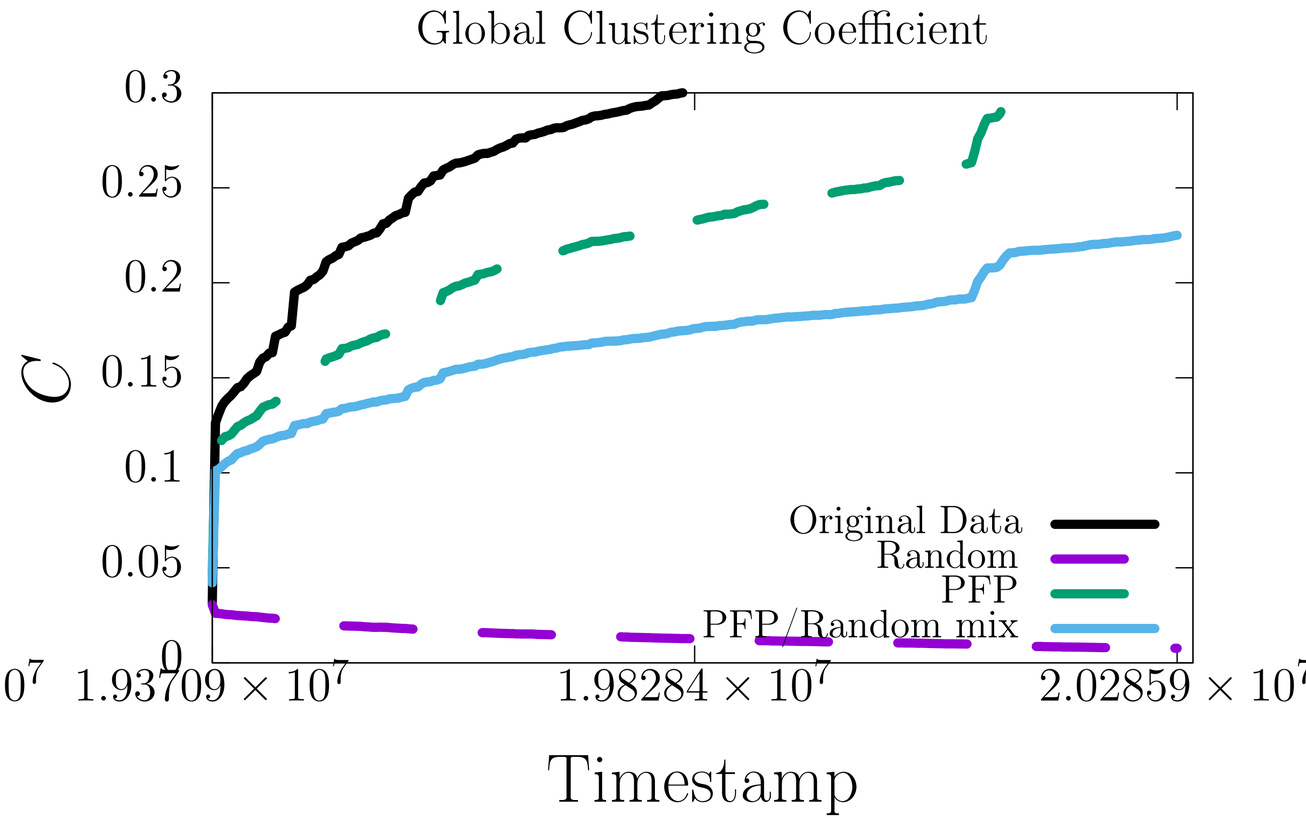

Remarks

- Often (**not always**) the highest likelihood mixture of model components may generate networks with better matching statistics than any single component alone

- But this highest likelihood mix is not guaranteed to be a good model - still need to look at network statistics

- Finding maximum likelihood mix of more than two model components may become expensive - candidate for ML techniques

Conclusions and future directions

- Temporal data allows calculation of likelihoods of dynamic models given observed network evolution
- With this measure we can distinguish models which generate networks that are similar in structure
- Fitting mixed models helps uncover the roles of different processes governing network growth
- Work in progress: using likelihood measure to analyse how such processes may change over time

Thank you for listening!
What are your questions?

github.com/narnolddd

n.a.arnold@qmul.ac.uk

@narnolddd